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Control of Integrated Voice/Data Multi-Hop Radio Networks Via Reduced-Load Approximations

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13. ABSTRACT *(Maximum 200 words)*

In this report we develop reduced-load approximation techniques based on the stochastic knapsack and the Pascal distribution, which enable the computationally efficient and accurate evaluation of the probability of voice blocking, the probability of data queueing, and the average data delay in integrated voice/data multi-hop radio networks. Monte-Carlo summation techniques are also used to verify the accuracy of the approximations. The reduced-load approximations exhibit excellent to very satisfactory accuracy for the entire range of network and traffic parameters of interest, while the computational effort necessary for their evaluation is substantially lower than that of the exact expressions (which is prohibitive for multi-hop radio networks of even moderate size). They are also used successfully to approximate the derivatives (sensitivities) of the above performance measures with respect to network and traffic parameters. These approximations are applicable to single-rate and multi-rate voice models, as well as to models with voice activity and silence periods. Besides the accurate and time-efficient performance evaluation of integrated voice/data multi-hop radio networks, the computational efficiency and the accuracy of these approximations enables their use for control and optimization purposes.

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CONTROL OF INTEGRATED VOICE/DATA MULTI-HOP RADIO NETWORKS VIA REDUCED-LOAD APPROXIMATIONS

1. INTRODUCTION

In this report, we address one of the major issues in multi-media networks: the combined (joint) admission control of voice calls and routing of data packets through the network. These two problems have been traditionally dealt with separately [1]-[3]. However, with the advent of ISDN and multi-media services [3], it has become necessary to design routing and call set up or scheduling schemes jointly, so that the network resources (bandwidth of links, processors at nodes) are used efficiently and all user requirements about the offered quality of services (QOS) of the different traffic types (e.g., data, voice, and video) are met.

One of the major obstacles in the joint optimization of voice admission control (or scheduling) and data routing schemes and, in general, of schemes which control admission, access, and routing for multi-media traffic, is the difficulty in obtaining (or the complete lack of) closed-form expressions for the performance measures characterizing QOS of the different traffic types.

In many practical situations where such optimizations are required, simplistic approximations of low accuracy are used to evaluate the performance measures of interest. These approximations neglect the interaction and interdependencies caused by the multi-hop network operation and the sharing of the network resources by the different traffic types. Consequently, the control schemes derived from the optimization of these coarsely approximated performance measures are sub-optimal and the network resources may not be utilized efficiently.

By contrast, the emphasis of this report lies in (i) the identification of appropriate methods for approximating accurately the performance measures involved in the problems of admission control of voice calls and of routing of data packets in integrated networks and (ii) the derivation of optimal schemes for admission control and routing on the basis of these approximations. In this context, some existing results on reduced-load approximations for voice traffic [4]-[10] are used, their applicability is extended from wired networks to wireless networks, and their accuracy is validated for a broader range of network and traffic parameters; in addition, new results for data traffic are derived for the first time. All approximations used are compared to each other and to confidence intervals of the

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actual performance measures derived via the Monte-Carlo summation method for a broad range of parameters of the traffic types.

Although the approximations and performance measures used in this report pertain to voice and data traffic and are used for the optimization of voice admission control and data routing schemes, they are in principle applicable to other problems (e.g., scheduling or set up of calls and data routing) and different traffic types (e.g., data, voice, and video). Actually, regarding the latter issue, our results cover multi-rate scenarios, according to which different traffic types may have different bandwidth requirements; thus, if voice but not video traffic is involved, voice traffic of different bit rates (and thus quality) can be accommodated. When video traffic is also present, our approach requires substantial modification to accommodate the variable rate traffic of video sources; however, it can be definitely extended to this case and thus it finds application to true multi-media scenarios.

Moreover, in our model of voice sources, we accommodate both periods of activity and silence. If the silent periods can be sensed by the network nodes, then the data users can, at least in theory, take advantage of this and utilize the released bandwidth, thus increasing the efficiency of the protocols. In practice, this monitoring of silence periods and talkspurts can be readily implemented only in certain situations and architectures and at the expense of channel bandwidth and additional complication in the network protocols; this issue is discussed further in Section 2.2.1.

Finally as elaborated in Section 12.4 the approximations of this report are also applicable (after suitable modification) to interesting problems of high-speed networks such as (i) call set-up and admission control in Asynchronous Transfer Mode (ATM) and (ii) multicasting of hierarchically encoded data.

The cost function that can be employed in the optimization of the voice admission control and data routing schemes consists of the weighted sum of

(i) the average blocking probabilities along the paths of voice calls

and

(ii1) the average probabilities of queuing of data along the links of the network or

(ii2) the average data delays along the links of the network

Closed-form expressions for the above quantities are either not available [as is the case for (ii1) and (ii2)], or, even when they are available [as is the case for the product form of (i)], they are very difficult to compute for moderate to large size networks. This difficulty is actually amplified by the fact that in optimization problems such as joint voice and data routing or voice admission control and data routing, the performance measures above may have to be evaluated repeatedly for several differents paths or links. It is exactly this difficulty that we attempt to circumvent in our work.

The **approach** followed in this report can then be summarized as follows. We use existing approximations or develop new ones for the average probability of voice blocking, the average probability of data queueing, and the average data-packet queueing data delay, and employ those instead of the exact expressions to derive near-optimal **admission control schemes based on thresholds** for the voice traffic (see Table 20). We do not derive optimal routing schemes for the data traffic in this report but since we approximate accurately the data revenue (and voice revenue) sensitivities with respect to the link capacities, the offered voice loads, and offered data loads, we can obtain near-optimal data routing schemes by using standard routing algorithms [2] based on these derivatives (sensitivities).

For **voice blocking probabilities**, we consider approximation methods suggested by Kelly [4]-[5] and the knapsack, Pascal, and Monte-Carlo summation approximation methods employed by Ross [6]-[9]. Mitra's approximation method [10] is also critically considered. All these approximations are known to be **asymptotically correct** (accurate) under a limiting regime, according to which both the capacity of the links of the radio network and the average input voice traffic (loads) increase to large values, while their ratio remains constant.

These approximation techniques are based on several distinct concepts and have varying degrees of accuracy and convergence range. **Kelly's approximation** [4]-[5] is based on an inter-link independence assumption for voice traffic. The **knapsack approximation** (used by Ross in [6] and [8]) is based on a stochastic knapsack concept. The **Pascal approximation** (used by Ross in [6] and [9]) is based on a birth-death process for modeling voice traffic that follows the Pascal distribution. The **Monte-Carlo summation method** employs acceptance/rejection methods used primarily in simulation techniques in order to evaluate multi-dimensional integrals under constraints [7]. This method generates confidence intervals for estimators of the actual performance measures of interest, and thus can be used to provide reliable means of comparison among all other approximations even when the exact expressions are not available or require prohibitive computational effort. **Mitra's approximation** [10] uses a Taylor series expansion on the normalization constant of the product form of the voice steady state probability distribution.

As part of our effort, the **accuracy and convergence range** of the knapsack and Pascal approximations were verified by comparing the approximate results with those obtained using the exact expressions (where feasible) or confidence intervals generated via the Monte-Carlo summation method. The agreement was found to be very satisfactory, not only for the average performance measures, i.e., the probabilities of voice blocking and data queueing (or the data queueing delay) when averaged over the traffic of all circuits or links, respectively, but also for similar performance measures corresponding to the individual circuits or links of the network.

Consequently, we use the knapsack approximation for the **optimization of the thresholds for the admission control** of voice calls. Besides the wired multi-rate loss networks of [6]-[10], we also considered radio networks modeled as in [11]-[12] where the transceivers at the nodes (rather than the link capacities) are the network resources. The application of these approximations to radio networks is new, and different technical problems than those of the multi-rate loss networks had to be addressed. However, the results based on Mitra's approximation were rather disappointing and will not be used in our optimization, despite the fact that we had extended the approach to general network topologies and multi-rate networks. A brief discussion of Mitra's approach and our assessment of its applicability to the problems considered in this report is presented in Section 4.4.

For the **probabilities of queueing** and the **queueing delays** of data, we use **Kleinrock's independence assumption for data traffic**; according to this assumption, the distribution of interarrival data packet times to the various internal nodes of the network remains exponential, even after the data packets have been serviced at intermediate links. Voice always maintains priority over data in our models. Due to the much longer average duration of typical voice calls compared to that of data packets (whose arrival process is characterized by a Poisson distribution), it is reasonable to assume that the voice state of the network changes much slower than that of the data state.

We can thus evaluate the probability of queueing of data (and the queueing delay) conditioned on the voice state (i.e., the number of active and inactive voice calls in the network); then we average with respect to the steady-state probability distribution of the voice state. Both M/M/c and M/D/c models for data queueing at the links have been considered. We have performed the aforementioned averaging of the M/M/c or M/D/c expressions (conditioned on the voice state) with respect to the voice state, according to the knapsack and Pascal approximation methods. This is a novel approach first appearing in this report. We limited attention to these data models because of constraints in the preparation time and the length of this report; the applicability of our approach is not restricted to these models, it can be applied (with proper modification) to any other data models as long as the assumption holds that changes in the state of the network voice traffic are much slower than those in the data traffic.

Finally, the **sensitivities** of suitably defined **voice and data revenue measures with respect to link capacities (or number of node transceivers), voice loads, and data loads** were evaluated via the knapsack approximation and shown to be very close to the actual values (refer to Tables 21 and 22). Again, these approximate sensitivities are much more computationally efficient than the cumbersome (and usually prohibitive) exact expressions. Actually, as our results establish, there will be almost negligible loss in revenue when voice-control schemes use these approximate sensitivities in place of the exact ones.

With the help of the aforementioned revenue sensitivities, important practical problems of **allocating additional network resources** in response to increasing voice and/or data network traffic demand can be easily handled with our approach, as well as problems of **data routing** in which the derivatives of the data delay (or the probability of queueing) are used by standard optimal routing algorithms. In this context near-optimal schemes for the **joint voice admission control and data routing** can be derived for both single-rate and multi-rate networks. This can also be accomplished for near-optimal schemes for **joint call set-up and data routing**. The range of applications of this methodology actually includes single-rate and multi-rate networks, wired or wireless (radio) networks, as well as high-speed networks.

It should be noted that, as summarized in Table 23, the computational effort required for the knapsack and Pascal approximations and the Monte-Carlo summation method compares very favorably with that necessary for the evaluation of the exact expressions. The reduced complexity permits the use of the approximations of this report for on-line optimization purposes.

1.1. Outline of the Report

The report is organized as follows. In Section 2, the network models, the source model for voice and data traffic, and the cost function together with the individual performance measures for voice and data of interest are described in detail. In Section 3, the steady state probability distribution of the voice state over the entire network is derived and the evaluation of the probability of voice-call blocking and of data-packet queueing is discussed. In Section 4, four approximation methods for evaluating the probability of voice blocking are reviewed, namely, Kelly's, knapsack, Pascal, and Mitra's. In Section 5, the knapsack approximation is extended and applied to the probability of data queueing. Subsequently, in Section 6 the Pascal approximation is extended and applied to the probability of data queueing. Following is Section 7 with the application of the knapsack and Pascal approximation techniques to the average data packet queueing delay. In Section 8, the Monte-Carlo Summation method is described in detail for the evaluation of the probabilities of voice blocking and data queueing. In Section 9, the knapsack approximation method is applied to voice admission control problems. In Section 10, the sensitivities of suitably defined voice and data revenue functions with respect to the link capacities, voice loads, and data loads are evaluated via the knapsack approximation. In Section 11, the various approximations are compared to each other and to confidence intervals generated via the Monte-Carlo summation method; the use of the approximations in obtaining near optimal thresholds for admission control is also described. Finally, in Section 12 several conclusions are drawn from this study.

2. SYSTEM MODEL FOR THE JOINT VOICE ADMISSION CONTROL AND DATA ROUTING PROBLEM

In this section we present the network and traffic models of interest in this study. A general purpose multi-hop multi-rate-voice/data network model and a voice/data multi-hop radio model are described first, followed by detailed models for the voice and data traffic, and by the definition of suitable performance measures.

2.1 Network Model

In both network models FDMA (frequency division multiple-access) is the multiplexing technique used; thus frequency channels (rather than time slots) are used to carry the packetized traffic (voice or data). Circuit-switching is the primary mode of communication for the network (for voice traffic), whereas packet-switching is used for data traffic.

2.1.1 General Multi-Hop Multi-Rate Network Model

The network we consider can be defined by a triplet $(\mathcal{N}, \mathcal{L}, \underline{c})$ where $\mathcal{N}$ is a set of nodes, $\mathcal{L}$ is the set of all possible directed links (each directed link l connects two nodes in $\mathcal{N}$) and $\underline{c} = [c_l, l \in \mathcal{L}]_{1\times|\mathcal{L}|}$ is the (row) vector containing the capacity (number of channels) c_l for each link $l \in \mathcal{L}$, where $|\mathcal{L}|$ denotes the number of elements in set $\mathcal{L}$. The set of consecutive links directed from source node n to destination node m constitutes the path p; we denote by $\mathcal{P}$ the set of all such paths; similarly $\mathcal{P}_l$ denotes the set of all paths that use link l; i.e.,

$$\mathcal{P}_l = \{p \in \mathcal{P} \mid l \in p\}, \ \ l \in \mathcal{L}$$

and is used frequently in our analysis. The routes followed by the network traffic are characterized by the $|\mathcal{L}| \times |\mathcal{P}|$ routing matrix $\mathbf{A}$ whose elements $A_{lp} = 1$, if the p-th path ($p \in \mathcal{P}$) uses link l ($l \in \mathcal{L}$), and $A_{lp} = 0$ otherwise.

2.1.2 Multi-Hop Radio Network Model

The notation is basically the same as in the previous section. However, in this model (motivated by the work of [11]), the number of transceivers at each node is the important resource instead of the capacity of the links of the previous section. Let T_n denote the number of transceivers at node n ($n \in \mathcal{N}$). The vector of node transceivers $\underline{T} = [T_n, n \in \mathcal{N}]_{1\times|\mathcal{N}|}$ replaces $\underline{c}$ defined above. Moreover, instead of $\mathcal{P}_l$ defined above, the set $\mathcal{P}_n$ defined as

$$\mathcal{P}_n = \{p \in \mathcal{P} \mid n \in p\}, \ \ n \in \mathcal{N}$$

and denoting the set of all paths p intersecting at node n appears frequently in our analysis.

2.2 Source Models

2.2.1 Multi-Hop Multi-Rate Network

We assume that the data packet and voice call arrival processes from outside the network with originating node n and destination node m are Poisson distributed with rates F^d_{nm} and F^v_{nm}, respectively. Moreover, F^d_l (F^v_l) denotes the data (voice) flow in link l, F^d_p (F^v_p) denotes the data (voice) flow input from outside the network to path p, μ^d_l is the data service rate on the l-th link ($l \in \mathcal{L}$), and μ^v_p is the voice service rate on the p-th path ($p \in \mathcal{P}$). The units of all these quantities (arrival and service rates) are packets per sec.

Since the network we are considering is of middle-size or larger and the data traffic loads are moderate to heavy, we may assume that Kleinrock's independence assumption holds for data. According to this approximation, which has been verified through simulations for networks with data-only traffic, the data-packet serial arrival process at each link, which includes both arrivals from outside the network, as well as packets forwarded by upstream nodes, can be accurately approximated with a Poisson process independent of the interaction taking place inside the network among the various nodes (queueing and servicing). For a multi-media network this assumption/approximation for the data traffic has not been verified but we expect it is valid for moderate to large size networks. In particular, we expect it to be valid for scenarios characterized by voice traffic that changes much slower than the data traffic; because as elaborated in Section 2.3 below, in these cases we can condition on the state of the voice traffic and work with conditional performance measures for the data traffic.

Two models are considered for data traffic. In the first, the packet length is exponentially distributed, resulting in an M/M/c queueing model, where c is the number of servers (channels) available for data. * The mean packet length is denoted as $1/\mu^d_{nm}$ (in secs). In the second, the packet length is a deterministic constant ($1/\mu^d_{nm}$), resulting in an M/D/c model.

The length of voice calls is exponentially distributed with parameter μ^v_{nm}. Actually, we assume that every call is composed of active and inactive (silent) periods which are exponentially distributed with parameters α and β (in sec^{-1}), respectively. The mean

* The number of channels available for data depends on the number of voice calls in progress.

duration of active periods is $1/\alpha$ (secs) and the mean duration of inactive periods is $1/\beta$. The speaker "activity fraction" is defined as $\frac{1/\alpha}{1/\alpha+1/\beta} = \frac{\beta}{\alpha+\beta}$ and typically takes the value 0.4 for normal conversational speech (in half-duplex mode). In full-duplex mode, the value 40% is valid for the activity of each of the source and destination nodes, in the sense that 40% of the time each node talks and another 40% of the time it listens to the other side talking. Therefore, the assigned circuit for the voice call remains occupied for 80% of the time; an intermediate node will be in transmit mode 80% of the time, i.e., 40% of the time transmitting in the "downstream" direction and 40% of the time transmitting in the "upstream" direction. The quantity $\beta/(\alpha+\beta)$, and its complement with respect to 1, i.e., the speaker silence fraction $\alpha/(\alpha+\beta)$, enter in the key expressions for the probability distribution of the state of voice calls in the network (see Section 3).

Under the model of the previous paragraph, the rate of active arriving calls is $F^v_{nm}\beta/(\alpha+\beta)$ and the rate of inactive arriving calls is $F^v_{nm}\alpha/(\alpha+\beta)$ * ; an active call turns inactive with rate α and a silent call becomes active with rate β.

We assume that the portion of the channel capacity (the time-varying number of channels c in the M/M/c and M/D/c queueing models) left unused by voice calls is used by data users. This represents a very desirable situation with the most efficient use of channel resources but not necessarily an easy one to achieve. It is required that the status of all voice conversations using a link is monitored and that this information is fed to the data users that have access to that link so that they can use the channel; they must however stop using the channel once the next talkspurt begins. If the silent period is less than a packet length and thus the data message must be interrupted by the resumption of the voice call, we may assume that the data message completes its packet transmission before the voice call resumes; since the packet length is so much smaller than the typical length of the voice conversation (or even the length of a talkspurt), the effect on the resumed voice conversation is anticipated to be negligible.

In systems that can not take advantage of voice call silent periods, all calls are assumed to be active throughout their duration. Under this model it is commonly accepted that the probability of blocking remains relatively insensitive to the specific form for the probability

* Although it would be more reasonable to assume that all calls are active when they arrive, the assumption that they may be in either the active or the silent mode simplifies the mathematical model, while resulting in little impact on numerical results.

distribution of the duration of voice calls [17] and it basically depends only on the mean of call length $1/\mu^v_{nm}$. With this simpler model the entire analysis is also simplified and can be obtained as a special case of the analysis for the two-state (active, inactive) voice call model presented in the following sections. This translation requires that we set $\alpha = 0$, $\beta = 1$, that the number of inactive calls denoted by n^s_p is set to 0 in all equations of the subsequent sections which involve it, and that we use the conventions $0! = 1$ and $0^0 = 1$.

It is assumed that the bandwidth of each of the channels comprising the links of the network is equal to the data rate (in bits per sec) of the data traffic and is obtained from the data packet arrival rate (in packets per sec). Thus the capacity c_l of any link l ($l \in \mathcal{L}$) represents the number of channels and is an integer. For voice traffic we consider single-rate and multiple-rate scenarios. In the single-rate case all voice calls have identical data rate (denoted by r) and require the same bandwidth for transmission. In the multi-rate case we assume that all voice sources using path p have data rate r_p as in [6]. The values of r and r_p used in the following sections are normalized with respect to the bandwidth of a single channel. These normalized r and r_p are not necessarily integers.

At this point let us clarify that by using the multi-rate traffic model of the previous paragraph we can model multi-class voice traffic. Thus, voice traffic of several different quality specifications (such as fully compressed, partially compressed, or uncompressed voice, secure voice etc.) and bandwidth requirements can be modeled. Indeed, all we need to do is to characterize the paths $p \in \mathcal{P}$ not only by the collection of consecutive links included in them, but, also by the (possibly) different data rates r_p of voice traffic that flow through them. In this way more than one element of $\mathcal{P}$ may follow the same physical path (route) inside the network but carry different amounts of information (have different data rates). Finally, the multi-class traffic of the above model need not be limited to voice only; traffic types requiring higher bandwidth than voice, such as video of different bandwidth requirements and rate variabilities [such as video telephony (teleconferencing) with medium high variable rate or full-motion video (television) with high variable rate] can also be dealt with in the same manner.

Since voice calls can be blocked but can not tolerate delay whereas data can be delayed, we assume that voice has priority over data. Data arrivals are routed in a packet-switching manner so that the input data flow in each node is separated into several subflows and each subflow takes a different path to its destination. Each voice call, after being admitted, is allocated a channel along a fixed multihop path to its destination in a virtual circuit manner, that is, once the path is chosen, the call uses only this path to transmit until the call is finished.

On the other hand, data messages (following a M/D/c_l' traffic model) can use only the residual capacity c_l', that is the portion of the link capacity c_l that remains unused after the active voice calls on paths employing link l have occupied the necessary number of channels. The appropriate expression for the data link capacity c_l' is provided in Section 2.4.1 below.

An alternative model that always guarantees that a portion of the capacity of each link is allocated to data communications is also considered in this report [refer to equation (7.17) of Section 7.3].

2.2.2 Single-Rate Multi-Hop Radio Network

The model for voice calls for the radio model of Section 2.1.2 is similar to the one described above, except that we address only the case in which a single-rate model for all calls is adopted, that is,

$$r_p = r = 1 \text{ for all } p \in \mathcal{P}.$$

Also data messages use the same data rate as voice calls.

For the data traffic we use an M/D/c_l' model where the number of data messages that can be transmitted simultaneously over link $l = (n, m)$ (connecting nodes n and m) is limited by the residual capacity c_l' of the link; that is, the remaining number of channels once the active voice calls have occupied the necessary transceivers at nodes n and m. The appropriate expression for the residual capacity is given in Section 2.4.2. Data packets are queued at buffers available at the nodes. The above M/D/c model assumption can not be fully justified here as it was done for the general wired network of Section 2.2.1., because of the need to coordinate the transceivers at the two nodes of each link. A radio network will most probably use some more complicated access protocol for the data traffic, resulting in arrivals to intermediate data links (or nodes) which are not Poisson. Therefore Kleinrock's independence assumption is less likely to be valid in a radio environment. However, the simple M/D/c model enables us to evaluate the accuracy of the approximations described in this report without having to evaluate complicated protocols for the data traffic. For different protocols than the M/D/c, certain conditional probabilities pertaining to data behavior [which are conditioned on the number of ongoing (active) voice calls] will have to be used instead of the M/D/c formulas; except for this change the basic steps of our approach are applicable to these cases as well. A more detailed discussion of this issue is provided at the end of Section 2.4.2.

An alternative model that always guarantees that some transceivers (and thus some link capacity) are dedicated to data traffic is also considered in Section 7.3. [refer to equations (7.18)-(7.19)].

2.3 Cost Function / Performance Measures

The performance measures of interest to our study are:

(i) **the probability of blocking of calls** B_p along each path p ($p \in \mathcal{P}$), as well as its average over the voice arrival process at all paths, $\bar{B}$,

(ii) the **probability of queuing data** Q_l at each link l ($l \in \mathcal{L}$), as well as its average over the data arrival process at all links, $\bar{Q}$,

(iii) and the **average queueing delay of data** W_l at each link l ($l \in \mathcal{L}$), as well as its average over the data arrival process at all links, $\bar{W}$.

The first of these quantities B_p is defined as the probability that an arriving call of class p (i.e., destined to follow voice path p) finds all available channels busy at one or more links on path p, and is therefore blocked. A precise mathematical definition of B_p is provided by equation (3.12b) of Section 3 as the quotient of two normalization constants. The quantity Q_l is the probability that a data packet arriving at link l finds all channels of that link busy and is queued, and W_l is the average delay at the queue of link l experienced by a typical data packet. Since W_l provides only the average value of the delay and the distribution of the delay is very difficult to obtain, we also provide the probability of queueing Q_l (which is easier to obtain) in order to supplement the information given by W_l about queueing at link l. Links are considered in isolation here because of Kleinrock's independence assumption, which we assume is valid for the data traffic; this is discussed in more detail below.

The aforementioned average quantities $\bar{B}$, $\bar{Q}$, and $\bar{W}$ are defined by

$$\bar{B} = \frac{\sum_{p\in\mathcal{P}} \rho_p^v B_p}{\sum_{p\in\mathcal{P}} \rho_p^v}, \tag{2.1a}$$

$$\bar{Q} = \frac{\sum_{l\in\mathcal{L}} \rho_l^d Q_l}{\sum_{l\in\mathcal{L}} \rho_l^d}, \tag{2.1b}$$

and

$$\bar{W} = \frac{\sum_{l\in\mathcal{L}} \rho_l^d W_l}{\sum_{l\in\mathcal{L}} \rho_l^d}, \tag{2.1c}$$

respectively, where the utilization factors for voice and data traffic (ρ_p^v and ρ_l^d), are defined in equations (2.7) and (2.8) later in this section.

The cost function C of interest to problems of joint admission control and data routing (or joint voice and data routing) consists of weighted sums of $\bar{B}$ and $\bar{Q}$ or of $\bar{B}$ and $\bar{W}$, that is,

$$C = K_1\bar{B} + K_2\bar{Q} \tag{2.2a}$$

or

$$C' = K_1\bar{B} + K_2\bar{W} \tag{2.2b}$$

where $K_1 + K_2 = 1$ and $K_1, K_2 \geq 0$. Therefore, in order to perform any optimization involving the above cost functions we need to have expressions (closed-form or accurate approximations) available for the probability of voice blocking ($\bar{B}$), the probability of data queueing ($\bar{Q}$), and the average queueing delay of data ($\bar{W}$). Since such computations are typically invoked many times during optimization the evaluation of the exact expressions (or the approximations) should be computationally efficient, otherwise optimization is not computationally feasible.

Let us revisit now B_p, the performance measure for the voice calls. Since voice calls have preemptive priority over data, the performance of voice calls is not affected by the data; consequently, the well-known product-form solution of the probability of blocking ([3], [4]) is valid. The difficulty in evaluating B_p of course lies in the computational complexity of the expression for the probability of blocking. Our approach here is to consider approximations to the probability of blocking that are asymptotically accurate under specific limiting regimes. Although B_p is of greatest interest in most applications, two additional voice blocking probabilities are also used: B_{lp} (approximations to it are denoted by L_{lp} in Section 4), the probability that the capacity of link l along the path of a voice call of class p ($p \in \mathcal{P}$) is not available [whose precise mathematical definition is given in (3.12a) as the quotient of two normalization constants]; and B_l, the probability of blocking of voice calls due to unavailability of the capacity of link l ($l \in \mathcal{L}$). Both B_p and B_l are approximated with the help of B_{lp}; details are provided in Section 4.

Next consider Q_l and W_l, the performance measures for data. Since the states of data at different links are dependent—for example, for all links on the path of a voice call the data states depend on the state of calls of the path and as such are mutually dependent—it is not possible to consider Q_l for link l without the coupling with the other links. To obtain Q_l ($l \in \mathcal{L}$) we have to consider the state of all links at a particular time, which is very difficult when the output data process of each link is not Poisson.

To facilitate the analysis of data we need an additional assumption (beyond Kleinrock's independence assumption). Since the state of voice calls changes much slower than the data states, we assume that data can reach steady-state during the sojourn time of calls within a particular state. The validity of this assumption has not been verified via simulation; in the future, we plan to check the accuracy of this approximation via simulation, at least for small-size networks with voice and data traffic. However, a similar assumption has been made in [14] for the analysis of Voice/Data (VD) Interleaved-Frame Fixed-Length (IFFL) protocols in the context of movable-boundary channel-access schemes for integrated voice/data networks. It was shown that this assumption is pessimistic at moderate to high throughput levels. VD-IFFL protocols work with time-slotted networks, and use reservations for the voice traffic and IFFL (which combines reservation with contention) for the data traffic. Under the above assumption and the aforementioned Kleinrock's assumption (discussed in detail in Section 2.2.1) we can examine each link in isolation and consider $M/M/c'$ or $M/D/c'$ models for the data conditioned on the state of the voice taking on a particular value.

2.4 Basic Notation

The notation introduced here is used in all subsequent sections. However, additional notation is introduced in each section as necessary. In this section attaching an "a" or a "b" to the number of an equation respectively signifies the single-rate or the multi-rate general (not necessarily radio) network scenarios; attaching a "c" signifies a single-rate radio network with transceivers at the nodes.

2.4.1 General Multi-Rate Multi-Hop Network

The system state is described by a triplet that contains global (networkwide) information in the voice state, along with local information on the data state at one particular link.

Denote the state as $(\underline{N}^v, \underline{N}^s, n_l^d)$, where

$\underline{N}^v = [n_p^v : n_p^v =$ number of calls taking path p in the system, $p \in \mathcal{P}]_{1\times|\mathcal{P}|}$

$\underline{N}^s = [n_p^s : n_p^s =$ number of calls in silent phase out of n_p^v calls, $p \in \mathcal{P}]_{1\times|\mathcal{P}|}$

$n_l^d \triangleq$ number of data messages at link l

Thus the number of ongoing voice calls along the paths (circuits) and the number of data messages along the links are the network states of interest to this report. System performance must be evaluated for each of the $\mathcal{L}$ links in the network. Links can be considered in isolation because of Kleinrock's interlink independence assumption (discussed at the beginning of Section 2.2.1) and the additional assumption about the state of voice varying much slower than that of data (cited at the end of Section 2.3) which decouples the variations of data traffic from voice traffic and gives meaning to Kleinrock's assumption for the conditional data traffic (conditioned on the voice state). We reiterate here that it is not necessary for the validity of our approach to consider the data links in isolation; we do it because it simplifies the computations (at least with respect to the data part) and allows us to test our approximations for performance measures (for data traffic) which are available in closed form (when conditioned on the voice state). Our approach can be extended to apply to performance measures (for data) which reflect the interaction between several data links; however, this requires additional computational complexity. This issue is discussed further in Section 2.4.2.

Note that $|\mathcal{P}|$ denotes the number of the elements in set $\mathcal{P}$, i.e., it is the number of call types and thus the number of paths, $\underline{N}^v$ and $\underline{N}^s$ are $|\mathcal{P}|$-dimensional vectors, and n_l^d is a scalar. Denote the set of all possible $(\underline{N}^v, \underline{N}^s, n_l^d)$ for the single-rate case as Ω, where

$$\Omega \triangleq \left\{ (\underline{N}^v, \underline{N}^s, n_l^d) \middle| 0 \le n_p^s \le n_p^v, 0 \le n_p^v \le c_p/r, p \in \mathcal{P}; \sum_{p\in\mathcal{P}_l} n_p^v \le c_l/r, \quad l \in \mathcal{L} \right\} \tag{2.3a}$$

where $c_p = \min\{c_l, \quad l \in p\}$ denotes the capacity of path p, and $\mathcal{P}_l$ was defined in Section 2.1.1. The terms c_p/r and c_l/r in (2.3a) represent the actual numbers of voice channels in c_p and c_l; recall that the link capacities c_l are normalized to the data traffic rate and thus represent the maximum number of data channels, and division by r (the voice data rate) is required in order to provide the corresponding number of voice channels.

Actually, the intuitive constraint $0 \le n_p^v \le c_p/r$ for $p \in \mathcal{P}$, which guarantees that the number of calls on path p does not exceed the (voice) capacity of path p defined as $c_p =$

$\min\{c_l, l \in p\}$, is satisfied whenever the more powerful constraint $\sum_{p \in \mathcal{P}_l} n_p^v \leq c_l/r, \;\; l \in \mathcal{L}$ is met. Therefore, the necessary (with the minimum number of constraints) form of Ω is

$$\Omega \triangleq \left\{ (\underline{N}^v, \underline{N}^s, n_l^d) \Big| 0 \leq n_p^s \leq n_p^v, p \in \mathcal{P}; \sum_{p \in \mathcal{P}_l} n_p^v \leq c_l/r, \;\; l \in \mathcal{L} \right\}. \tag{2.3a'}$$

Note that in the multi-rate case Ω becomes

$$\Omega \triangleq \left\{ (\underline{N}^v, \underline{N}^s, n_l^d) \Big| 0 \leq n_p^s \leq n_p^v, 0 \leq r_p n_p^v \leq c_p, p \in \mathcal{P}; \sum_{p \in \mathcal{P}_l} r_p n_p^v \leq c_l, \;\; l \in \mathcal{L} \right\}. \tag{2.3b}$$

The following additional notation is used in subsequent sections. In the single-rate case

$$n_l^v \triangleq \sum_{p \in \mathcal{P}_l} n_p^v \tag{2.4a}$$

$$n_l^s \triangleq \sum_{p \in \mathcal{P}_l} n_p^s \tag{2.5a}$$

denote the total number of calls and "silent" calls, respectively, that use link l; both n_l^v and n_l^s are clearly integers and satisfy the inequalities

$$0 \leq n_l^s \leq n_l^v \leq c_l.$$

The corresponding definitions in the multi-rate case are

$$k_l^v \triangleq \sum_{p \in \mathcal{P}_l} r_p n_p^v \tag{2.4b}$$

$$k_l^s \triangleq \sum_{p \in \mathcal{P}_l} r_p n_p^s \tag{2.5b}$$

where k_l^v and k_l^s now denote the total number of channels occupied by all calls and by the inactive (silent) calls, respectively, on link l; clearly these must satisfy

$$0 \leq k_l^s \leq k_l^v \leq c_l.$$

Since, the voice data rates r_p may not be integers, k_l^v and k_l^s may assume non-integer values as well.

Moreover, n_l^d (the number of data messages at link l) follows an M/D/c_l' model with residual data capacity

$$c_l' = c_l - (n_l^v - n_l^s) \tag{2.6a}$$

for the model of (2.3a) or

$$c_l' = c_l - (k_l^v - k_l^s) \tag{2.6b}$$

for the model of (2.3b).

Finally, since F and μ appear in the form $\rho = F/\mu$ in all the formulas derived hereafter, we can simply use the variable

$$\rho_p^v = F_p^v/\mu_p^v \tag{2.7}$$

$$\rho_l^d = F_l^d/\mu_l^d \tag{2.8}$$

$$\rho_l^v = \sum_{p \in \mathcal{P}_l} \rho_p^v \tag{2.9}$$

$$\rho_{nm}^v = F_{nm}^d/\mu_{nm}^v \tag{2.10}$$

$$\rho_{nm}^d = F_{nm}^d/\mu_{nm}^d \tag{2.11}$$

and

$$\rho_{nm}^v = \sum_{p \in \mathcal{P}_{nm}} \rho_p^v. \tag{2.12}$$

2.4.2 Single-Rate Multi-Hop Radio Network

In this model, the link capacities c_l ($l \in \mathcal{L}$) are replaced by the number of transceivers at node n, T_n ($n \in \mathcal{N}$). The key sets of paths $\mathcal{P}_l$ ($l \in \mathcal{L}$) are replaced by the sets $\mathcal{P}_n$, $n \in \mathcal{N}$. Now the number of calls n_p^v of class p (along path p, $p \in \mathcal{P}_n$) satisfies the constraints

$$\sum_{p \in \mathcal{P}_n} n_p^v \leq T_n, \quad n \in \mathcal{N} \tag{2.3c}$$

instead of (2.3a) or (2.3b), and the residual data capacity of link $l = (n, m)$ connecting nodes n and m is

$$c_l' = \min\left\{ T_n - \sum_{p \in \mathcal{P}_n} (n_p^v - n_p^s), T_m - \sum_{p \in \mathcal{P}_m} (n_p^v - n_p^s) \right\}, \ l \in \mathcal{L}, \ n, m \in \mathcal{N} \tag{2.6c}$$

This formulation implicitly assumes that all transceivers at nodes n and m that are not currently supporting active voice calls are available to support data traffic between nodes n and m. Thus it neglects the possibility of data traffic between either of these nodes and their other neighbors. Therefore, Eq. (2.6c) should be accepted with reservation and only as representative of the simplest data model that we can handle with our approach.

In a more realistic scenario the performance measure for the data operating under a particular protocol over link $l = (n, m)$ (connecting nodes n and m) depends not only the T_n and T_m of these nodes and the total number of active voice calls $\sum_{p \in \mathcal{P}_n}(n_p^v - n_p^s)$ and $\sum_{p \in \mathcal{P}_m}(n_p^v - n_p^s)$ but also on the corresponding values of these quantities for all the neighbors of the nodes n and m. Thus the residual capacity $l = (n, m)$ depends on the way in which the unused transceivers at nodes n and m are allocated to support data links with their other neighbors as well.

To determine this residual capacity one must very carefully enumerate all possible pairs of transceivers that are occupied by voice traffic for each voice state, and then use a (not yet defined) protocol to determine how the unused transceivers are to be allocated to support data traffic between the pairs of users. Typically, a transceiver may be paired with several of its neighbors (one at a time) to form a link-activation schedule, in which case the data-traffic queueing model will have to be revised to reflect the fact that the server is not always available (as in a queueing system with vacations). As long as this allocation depends only on the voice state (i.e., not on the data queue sizes at each node), our formulation that addresses the data state at each link in isolation remains valid.

Once this allocation is made, the performance measure of interest can be thought as a functional which depends on several terms of the form $\sum_{p \in \mathcal{P}_{\hat{n}}}(n_p^v - n_p^s)$ where $\hat{n}$ is an immediate neighbor of n or m. If the number of neighbors of the nodes n and m is small then we can use the approach of Section 9.2 (based on additional conditioning) to average the performance measure for data with respect to the steady state distribution of the voice state vector. If the number of neighbors of n and m is larger, we may use the approach of Section 9.3 (based on fictitious links) to carry out this averaging.

In this report the M/D/c model of (2.6c) is sufficient for establishing the accuracy and computational efficiency of the knapsack and Pascal approximation techniques for a first application to integrated voice/data radio networks. In the future we will apply these techniques to more complicated data protocols.

3. STEADY-STATE DISTRIBUTION FOR THE SYSTEM STATE

In this section we provide the steady-state distribution of the state of the network, that is of $(\underline{N}^v, \underline{N}^s, n_l^d)$, for the single-rate and multi-rate cases under the assumption that the the state of voice calls changes much slower than the state of data.

Proposition 3.1. When the state of voice calls changes much slower than the data-packet state, the steady state probability of $(\underline{N}^v, \underline{N}^s, n_l^d)$ can be closely approximated by

$$P(\underline{N}^v, \underline{N}^s, n_l^d) = P(\underline{N}^v, \underline{N}^s) \cdot P(n_l^d | \underline{N}^v, \underline{N}^s) \tag{3.1}$$

where

(a) the steady state probability of the voice is

$$P(\underline{N}^v, \underline{N}^s) = \frac{1}{G} \cdot \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}} \tag{3.2}$$

and

$$G = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega^v} \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}} \tag{3.3}$$

for

$$\Omega^v = \begin{cases} \left\{ (\underline{N}^v, \underline{N}^s) \middle| \; 0 \le n_p^s \le n_p^v, p \in \mathcal{P}; 0 \le n_l^v \le c_l, \;\; l \in \mathcal{L} \right\} & \text{single} - \text{rate case} \\ \left\{ (\underline{N}^v, \underline{N}^s) \middle| \; 0 \le n_p^s \le n_p^v, p \in \mathcal{P}; 0 \le k_l^v \le c_l, \;\; l \in \mathcal{L} \right\} & \text{multi} - \text{rate case} \end{cases} \tag{3.4}$$

(b) the conditional steady state probability of the data for an M/M/c model is

$$P(n_l^d | \underline{N}^v, \underline{N}^s) = \begin{cases} P_0 \frac{(\rho_l^d)^{n_l^d}}{n_l^d!}, & n_l^d \le c_l' \\ P_0 \frac{(\rho_l^d)^{n_l^d} c_l'^{c_l'}}{c_l'! c_l'^{n_l^d}}, & n_l^d > c_l' \end{cases} \tag{3.5}$$

where

$$P_0 = \left[\sum_{n=0}^{c_l'-1} \frac{(\rho_l^d)^n}{n!} + \frac{(\rho_l^d)^{c_l'}}{c_l'! \left(1 - \frac{\rho_l^d}{c_l'}\right)} \right]^{-1}, \tag{3.6}$$

$\rho_p^v = F_l^p/\mu_p^v$, $\rho_l^d = F_l^d/\mu_l^d$, and

$$c_l' = \begin{cases} c_l - n_l^v + n_l^s \geq 1 & \text{single} - \text{rate case} \\ c_l - k_l^v + k_l^s \geq 1 & \text{multi} - \text{rate case} \end{cases} \tag{3.7}$$

Note that

$$P(n_l^d = \infty | \underline{N}^v, \underline{N}^s) = 1 \ \text{ when } \ c_l' = 0, \ \text{ or } \ c_l' \geq 1 \text{ but } \rho_l^d/c_l' \geq 1 \tag{3.8}$$

i.e., the data-packet queue becomes infinite when there is no residual capacity for data or when the offered data load of the link exceeds the residual link capacity.

(c) the conditional steady state probability of the data for an M/D/c model can not be obtained in closed form but can be approximated with arbitrary accuracy using Tijms' iterative algorithm (see [13]) on a M/D/c_l' system (recall $c_l' = c_l - k_l^v + k_l^s$); the procedure is sketched in Appendix B.

Proof: The proof of the main result (a) is provided in Appendix A.

Comments:

1. Note that the expression for $P(\underline{N}^v, \underline{N}^s)$ in Eq. (3.2) is a modified version of the traditional product form for the steady state probability of the voice state. In the more frequently studied case, in which the occurrence of silent periods is not addressed, we have (see for example [4] or [6])

$$P(\underline{N}^v) = \frac{1}{G} \cdot \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^v!} \tag{3.9}$$

where

$$G = \sum_{\underline{N}^v \in \Omega^v} \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^v!} \tag{3.10}$$

is the normalization constant associated with the state space

$$\Omega^v = \left\{ \underline{N}^v \middle| \ 0 \leq n_p^v p \in \mathcal{P}; 0 \leq \sum_{p \in \mathcal{P}_l} r_p n_p^v \leq c_l, \ \ l \in \mathcal{L} \right\}. \tag{3.11}$$

The reason that we need $P(\underline{N}^v, \underline{N}^s)$ instead of the usual $P(\underline{N}^v)$ is that we assume that silence periods of voice calls can be detected and the data traffic can use the released bandwidth, thus increasing the overall efficiency of the resource allocation.

2. Under the M/M/c data model and a single-rate network, $(n_l^d|\underline{N}^v, \underline{N}^s)$ corresponds to an $M/M/c_l - n_l^v + n_l^s$ system when $c_l' = c_l - n_l^v + n_l^s \geq 1$ and $\rho_l^d/c_l' < 1$, whereas $Pr(n_l^d = \infty|\underline{N}^v, \underline{N}^s) = 1$ when $c_l' = 0$ or $c_l' \geq 1$ but $\rho_l^d/c_l' \geq 1$. The same is valid for $(n_l^d|\underline{N}^v, \underline{N}^s)$ under an M/D/$c_l - n_l^v + n_l^s$ data model. For the multi-rate case we only need to replace n_l^v and n_l^s by k_l^v and k_l^s in the above comments.

3. The **probability of voice blocking** at link l of a call of class p (B_{lp}) can be expressed ([4]-[6]) as the ratio of two normalization constants (also termed partition functions) G defined in (3.10); specifically, the probability that such blocking does not occur is

$$1 - B_{lp} = \frac{G(c_1, c_2, \cdots, c_{l-1}, c_l - r_p, c_{l+1}, \cdots, c_{\mathcal{L}})}{G(c_1, c_2, \cdots, c_{l-1}, c_l, c_{l+1}, \cdots, c_{\mathcal{L}})} \tag{3.12a}$$

where the normalization constants G of (3.10) are denoted as functions of the $1 \times |\mathcal{L}|$ row vector of link capacities $\underline{c}$. The right-hand side of (3.12a) represents the steady-state probability of the vector of the number of voice calls over all paths of the network, when a single (voice) channel is removed from link l (and thus its capacity decreases to $c_l - r_p$). This is indeed the probability of no blocking because a new arriving call (requiring bandwidth r_p) can be accommodated by the link capacity. The evaluation of B_{lp} in (3.12a) requires the computation of the normalization constant G of (3.10). This may be a very computationally demanding task even for moderate size networks, because of the large number of points in the state space over which the summation is performed. Therefore, accurate and computationally efficient approximations are necessary. Several such approximations have been developed under different conditions (limiting regimes) (see [4]-[10]). In Section 4 we review some of them and select to work with two: the knapsack and the Pascal approximations, which we extend and modify (in Sections 5 and 6) so that they apply to performance measures pertaining to the data such as the average probability of queueing and the queueing delay of data at the links of the network.

Finally, B_p, the blocking probability for a call of type p ($p \in \mathcal{P}$, [4]-[6]), can be evaluated from the expression

$$1 - B_p = \frac{G(\underline{c} - r_p \underline{A}_p^T)}{G(\underline{c})} \tag{3.12b}$$

where $\underline{c}$ was defined above and $\underline{A}_p^T$ is the transpose of the p-th column (a $|\mathcal{L}| \times 1$ vector) routing matrix **A** defined at the beginning of Section 2. Equation (3.12b) is similar to that in (3.12a) with one major difference; in (3.12a) only the capacity of link l has been reduced by r_p (corresponding to availability of channel capacity along link l for an additional voice call) in the argument of the normalization constant G in the numerator; in (3.12b) the capacities of all links l used by path p ($l \in p$) have been reduced by r_p (corresponding to availability of channel capacity along all links l along path p for an additional voice call). The expression in (3.12b) requires about the same level of computational complexity as that of (3.12a); this becomes prohibitive for even moderate size networks. However, the approximations obtained for B_{l_p} in Section 4 can also be used (with proper modification) for B_p as well.

4. The **probability of data queueing** at link l is given by

$$\begin{aligned}Q_l &= Pr(\text{data queued in link } \ l) \\ &= Pr(c_l - n_l^v + n_l^s \le n_l^d) \\ &= 1 - Pr(c_l - n_l^v + n_l^s > n_l^d \ \text{ where } \ (n_l^v, n_l^s) \neq (c_l, 0)) \\ &= 1 - \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega^v} \left[\sum_{n_l^d = 0}^{c_l' - 1} I[(n_l^v, n_l^s) \neq (c_l, 0)] P(n_l^d | \underline{N}^v, \underline{N}^s) \right] P(\underline{N}^v, \underline{N}^s) \qquad (3.13)\end{aligned}$$

for a single-rate system; for a multi-rate system we must only replace the n_l^v and n_l^s by k_l^v and k_l^s defined above, respectively, in the above equation; I denotes the indicator function taking value 1 when its argument is true and 0 if it is false. Consequently, Q_l can be evaluated directly from the conditional steady state probabilities of the data $P(n_l^d | \underline{N}^v, \underline{N}^s)$ and the steady state probability of the voice $P(\underline{N}^v, \underline{N}^s)$. However, the computational complexity of the summation involved in the definition of $P(\underline{N}^v, \underline{N}^s)$ [see (a) of Proposition 3.1] coupled with the additional summation necessary for obtaining Q_l becomes prohibitive for large $|\mathcal{P}|$. This computation is carried out through computationally efficient approximations in Sections 5 and 6.

5. The **queueing delay of data** at link l can also be evaluated (actually approximated) using the basic results of this section. The details are provided in Section 7.

6. The results of this section were derived for a general multi-rate multi-hop wired network; they are valid for the single-rate multi-hop radio network of Sections 2.1.2 and 2.2.2 after we replace c_l ($l \in \mathcal{L}$) with T_n ($n \in \mathcal{N}$), $\mathcal{P}_l$ with $\mathcal{P}_n$, and set $r_p = 1$ for all $p \in \mathcal{P}$.

4. REDUCED-LOAD APPROXIMATIONS FOR THE VOICE BLOCKING PROBABILITY

As discussed in the comments following Proposition 3.1 in Section 3, the evaluation of closed form expressions for the probability of voice blocking at link l or along path p require substantial computer resources and time. In particular, when these expressions must be computed numerous times (i.e., for different control policies), as is the case in a variety of optimization problems including admission control and routing, it becomes imperative that computationally efficient approximations are developed that exhibit satisfactory accuracy and allow the speedy evaluation of these quantities.

In this section we describe a number of approximations that can be used to evaluate the probability of voice blocking with computational complexity considerably lower than that of the brute force approach. These are termed "reduced-load" approximations, and have been applied to multi-rate loss networks by a number of researchers [4]-[10] with very encouraging results. The term reduced-load pertains to a reduced- (or thinned-) load approximation of the traffic from all paths using a particular link, because of traffic blocked at other links, and is discussed in detail in Section 4.2. The approximations are known to be asymptotically correct in the limiting regime characterized by heavy offered traffic loads and large link capacities. These approximations had not been applied to data traffic analysis before our work in this report.

In the context of the advancement of this approximation theory and its applications to practical multi-media network problems, our main contribution elaborated upon in this report is fourfold. First, we establish that two of these approximation techniques (knapsack and Pascal) exhibit satisfactory accuracy even when applied to situations different from those of the limiting regime; we actually show that they maintain excellent accuracy over the entire range of useful traffic scenarios and architectures. Second, we show how to use these approximations to evaluate performance measures for data (such as the probability of queueing for data and the average queueing delay for data) in networks with multi-media traffic. Third, we extend the application of these approximations to radio network models that are distinctly different from the multi-media networks of [4]-[10], which use optical fiber or copper as the transmission medium. Fourth, we apply these techniques to systems with admission control schemes and to the subsequent optimization of the thresholds or other control parameters involved.

The purpose of reviewing these approximation techniques here is twofold. First, as we show in Section 11 through comparisons with results based on the exact expressions and the Monte-Carlo summation method, these techniques have satisfactory accuracy and we can use them to approximate the **average voice blocking probability** of links or paths (routes) for radio networks of interest to our project. Second, we need to introduce, motivate, and describe the fundamental principles of these techniques before we extend and modify them in order to approximate the **average probabilities of queueing** and the **average waiting delays of data** at the links of the network. These extensions are described in detail in Sections 5, 6, and 7, and enable us to approximate accurately and with reasonable computational complexity important performance measures for the data in multi-media networks.

In this section we describe the principle of four of these approximation techniques: Kelly's, Knapsack, Pascal, and Mitra's approximations. We limited our consideration to these four for two reasons. First, these four are the ones that have been applied most successfully to a variety of networking problems with very satisfactory results. Second, we were able to extend and modify two of those (the knapsack and Pascal approximations) to accommodate performance measures for the data (such as average probability of queueing and average queueing delay). Kelly's approximation ([4]-[5]) was included in this review because it is a useful starting point for introducing the notation and the principle of the reduced load approximations. Mitra's approximation was included because we considered it early during the course of this work, we extended its application to more general topologies than the tree network of the original paper [10], and we extended and modified it so that it applies also to the probability of queueing and the queueing delay of data. Unfortunately, as we found out through comparisons with the exact expressions and the Monte-Carlo summation method, this approximation is accurate (actually converges) only under rather restrictive assumptions about the loads of the links and thus it is not presented in greater detail in this work.

4.1 Kelly's Approximation

Consider a network supporting multi-rate traffic (say with data rate r_p for path $p \in \mathcal{P}$). Denote by L_l the approximate probability that "all circuits are busy on the link l", or equivalently the probability of blocking for link l ($l \in \mathcal{L}$). Under the assumption ([4]-[5]) that these events occur independently from link to link, class p connections (voice calls using path p) arrive to link l according to a Poisson process with offered load $r_p \rho_p^v \prod_{\ell \in p, \ell \neq l}(1 - L_\ell)^{r_p}$ and thus the total arriving traffic at link l (belonging to all classes $p \in \mathcal{P}$) is also Poisson with aggregate load

$$\sum_{p \in \mathcal{P}_l} r_p \rho_p^v \prod_{\ell \in p, \ell \neq l} (1 - L_\ell)^{r_p}. \tag{4.1}$$

where the expression $\prod_{\ell \in p, \ell \neq l}(1 - L_\ell)^{r_p}$ represents the "thinning" or "reduced load" effect associated with the blockage of calls of type p at the other links along the path. Hence, under the link independence assumption, we must have

$$L_l = E\left[c_l; \sum_{p \in \mathcal{P}_l} r_p \rho_p^v \prod_{\ell \in p, \ell \neq l} (1 - L_\ell)^{r_p}\right], \quad l \in \mathcal{L} \tag{4.2}$$

where

$$E[c; \rho] = \frac{\rho^c / c!}{\sum_{n=0}^{c} \rho^n / n!} \tag{4.3}$$

is the Erlang loss formula (see [1] or [4]). In the above notation we used L_l instead of B_l to distinguish between the exact value of the voice blocking probability and the reduced-load approximation L_l. If $r_p = r$ for all $p \in \mathcal{P}$, then (4.2) becomes the standard reduced load approximation for single-rate loss networks.

Repeated substitutions are often used for finding a solution $(L_1, L_2, \cdots, L_{|\mathcal{L}|})$ to the fixed-point equation (4.2). Although oscillation can occur in (4.2), repeated substitutions typically converge to a fixed point for networks of practical interest. One of the features of this approximation scheme is that (4.2) has a unique fixed-point solution. The proof of uniqueness relies on the monotonicity properties of the Erlang loss formula; unfortunately, these properties are not possessed by the knapsack and Pascal approximation schemes discussed below.

Once a fixed point is found, the probability B_p that a call is blocked along path p (or equivalently a class-p connection is blocked as we saw in the discussion of Section 2.2) can be approximated by

$$B_p = 1 - \prod_{l \in p} (1 - L_l)^{r_p}, \tag{4.4}$$

a formula that invokes again the link independence assumption.

The above approximation is asymptotically accurate (correct) under the following limiting regime: the ratio of offered load and of the number of channels in each link is held fixed while the individual values of these two quantities become asymptotically large.

4.2 Knapsack Approximation

For this approximation scheme as well as the one of the following section we work first with a single-link multi-rate system and then extend the result to multi-link networks.

This technique is termed the knapsack approximation because the single-link multi-rate system corresponds to a ***stochastic knapsack*** resembling the knapsack model in combinatorial optimization. The term stochastic knapsack is motivated by the fact that typically the system modeled resembles a knapsack to which items (states) are added or from which items are taken out according to a probability distribution. This approximation was successfully applied to circuit-switching problems by Ross [6]-[9].

Consider a **single-link** system with link capacity c_l, which supports classes $p \in \mathcal{P}_l$ (or equivalently several paths p use link l) with data rates r_p and offered loads ρ_p. The probability that a class-p connection is blocked (or a voice call along path p is blocked), when arriving to the stochastic knapsack, is given by

$$K_{lp}[c_l; \rho_q^v,\ q \in \mathcal{P}_l] = 1 - \frac{\sum_{n=0}^{c_l - r_p} w(n)}{\sum_{n=0}^{c_l} w(n)}, \quad p \in \mathcal{P}_l \tag{4.5}$$

where

$$w(n) = \frac{1}{n} \sum_{q \in \mathcal{P}_l} r_q \rho_q^v w(n - r_q), \quad n = 1, 2 \ldots, c_l \tag{4.6a}$$

with initial condition

$$w(0) = 1. \tag{4.6b}$$

The intuitive explanation of (4.5) is that $1 - K_{lp}$ represents the probability of no blocking, i.e., the probability that up to $c_l - r_p$ ($l \in p$) voice calls are in the system (inside the stochastic knapsack) so that an arriving call (requiring r_p channels) can be accommodated by the capacity of link l without being blocked. In (4.5) $w(n)$ provides the probability that n voice calls are currently in the system, and the recursion (4.6a) for the update of $w(n)$ is typical of stochastic knapsack system models.

For a multi-rate loss network with **multiple links**, we denote by L_{lp} the approximate probability that "less than r_p channels are available on link l" (probability of voice blocking on link l along path p); L_{lp} is an approximation to the quantity B_{lp} defined by (3.12a) in Section 3. As in Kelly's approximation, we again assume here that these events occur independently from link to link. This approximation decouples the blocking phenomena on different links and enables the evaluation of an approximate expression for the amount of unblocked traffic traveling through the network and finally for the desired blocking probability. Under the interlink blocking independence assumption, class-q connections arrive to link l according to a Poisson process with offered load

$$\rho_q^v \prod_{\ell \in q, \ell \neq l} (1 - L_{\ell q}). \tag{4.7}$$

This load is termed thinned or reduced, because it is smaller than the corresponding load ρ_q^v (from circuit p) of the link l (for $l \in q$), when considered in isolation, in a manner that reflects the effect of blocking at the other links in the network through the approximate probabilities $L_{\ell q}$. This aspect of the approach gives the name reduced-load approximation to the knapsack approximation method (as well as the Pascal approximation method described in Section 4.3). Consequently, under the link independence assumption, we have

$$L_{lp} = K_{lp}[c_l; \rho_q^v \prod_{\ell \in q, \ell \neq l} (1 - L_{\ell q}),\ q \in \mathcal{P}_l], \quad p \in \mathcal{P}_l,\ l \in \mathcal{L}. \tag{4.8}$$

Equations (4.8) define a continuous mapping from the compact convex set $[0,1]^{|\mathcal{P}_1| \times |\mathcal{P}_2| \times \cdots \times |\mathcal{P}_{|\mathcal{L}|}|}$ into itself; thus, by the Brouwer fixed-point theorem, there exists a solution (L_{lp}, $p \in \mathcal{P}_l$, $l = 1, 2, \cdots, |\mathcal{L}|$) to (4.8). The method of successive approximations (i.e., repeated substitutions) can be employed to find such a solution. Once a solution to (4.8) has been obtained, the probability of blocking a class-p connection can be approximated by

$$B_p = 1 - \prod_{l \in p} (1 - L_{lp}) \tag{4.9}$$

and the probability of blocking any voice call on link l can be approximated by

$$B_l = \sum_{p \in \mathcal{P}_l} L_{lp}. \tag{4.10}$$

In contrast to the fixed-point equation (4.2) the solution to (4.8) of the knapsack approximation is not necessarily unique. This is because the knapsack equations (4.5)-(4.6) do not have the nice monotonicity properties of the Erlang formula (4.3). Multiple solutions to the fixed-point equations of (4.8) can alert the designer of potential instabilities in the network. Ross [6] provides an example where the network alternates between long periods of carrying only narrowband connections and long periods of carrying only wideband connections. Kelly's approximation does not expose this instability, since it always gives rise to a unique fixed point. But the knapsack approximation gives one solution with almost 100% blocking of narrowband connections and another solution with almost 100% blocking of wideband calls, which reflects the instability that actually exists in the system.

The knapsack approximation is shown in [6] and [8] to be asymptotically correct under the same limiting regime as Kelly's approximation (see end of Section 4.1). However, comparison of the accuracies of the two approximations, the exact expressions, and the Monte-Carlo summation method indicate that the knapsack approximation maintains satisfactory accuracy even far outside the limiting regime.

4.3 Pascal Approximation

The Pascal approximation technique is based on the use of a birth-death process with Pascal distribution to model the voice state in the system. This was applied to circuit-switched problems by Ross [6]-[9].

We again address first the case of a single link. Consider a birth-death process on the state space $0, 1, \cdots, c_l$, which, when in state n, has a death rate of n and a birth rate of $\epsilon^2/\sigma^2 + n(1 - \epsilon/\sigma^2)$, where ϵ and σ^2 are given positive numbers. Let $q(n)$, for $n = 0, 1, \cdots, c_l$, be the equilibrium probability of being in state n, that is, $q(n)$ satisfies

$$nq(n) = [\epsilon^2/\sigma^2 + (n-1)(1 - \epsilon/\sigma^2)]q(n-1), \quad \text{for } n = 1, 2, \cdots, c_l \tag{4.11a}$$

where

$$\sum_{n=0}^{c_l} q(n) = 1. \tag{4.12a}$$

Denote

$$P_{lp}(c_l; \epsilon; \sigma^2) = \sum_{n=c_l-r_p+1}^{c_l} q(n) = 1 - \sum_{n=0}^{c_l - r_p} q(n). \tag{4.13a}$$

The right-hand side of (4.13a) represents the probability that the birth-death process n is in a state $> c_l - r_p$. This corresponds to blocking since there is no room left in the capacity of link l for accommodating any arriving voice call (which requires r_p channels).

When $c_l = \infty$, $q(n)$ has the Pascal distribution, and the means and variance of the birth-death process are given by ϵ and σ^2, respectively (see [6]). The Pascal approximation uses the same approach but for finite values of c_l necessitating the solution of Eq. (4.11a), as described below.

If instead of $q(n)$ we use the normalized version

$$q'(n) = q(n)/q(0)$$

the recursion of (4.11a) becomes

$$q'(n) = \frac{1}{n}[\epsilon^2/\sigma^2 + (n-1)(1-\epsilon/\sigma^2)]q'(n-1), \quad \text{for } n = 1, 2, \cdots, c_l \tag{4.11b}$$

with initial condition

$$q'(0) = 1. \tag{4.12b}$$

This is easier to evaluate [it does not require knowledge of $q(0)$], and results in

$$\left[\sum_{n=0}^{c_l} q'(n)\right]^{-1} = q(0).$$

Using this result to modify (4.13a) yields

$$P_{lp}(c_l; \epsilon; \sigma^2) = 1 - \frac{\sum_{n=0}^{c_l - r_p} q'(n)}{\sum_{n=0}^{c_l} q'(n)}. \tag{4.13b}$$

Next consider the stochastic knapsack model, discussed in the previous subsection, for the isolated link l. When $c_l = \infty$, the mean and variance of the number of busy circuits [under the knapsack approximation for the steady-state voice distribution] are obtained after some manipulations by

$$\epsilon_l = \sum_{q \in \mathcal{P}_l} r_q \left[\sum_{n=0}^{\infty} n P(n_q^v = n)\right] = \sum_{q \in \mathcal{P}_l} r_q \rho_q^v \tag{4.14}$$

and

$$\sigma_l^2 = \sum_{q \in \mathcal{P}_l} r_q^2 \left[\sum_{n=0}^{\infty} (n - \rho_q^v)^2 P(n_q^v = n)\right] = \sum_{q \in \mathcal{P}_l} r_q^2 \rho_q^v \tag{4.15}$$

respectively. Thus, the infinite-capacity stochastic knapsack gives the same mean and variance for the number of busy circuits as the birth-death process with parameters $\epsilon = \epsilon_l$ and $\sigma^2 = \sigma_l^2$. It is, therefore, natural to approximate the probability of blocking a class-p connection arriving to the stochastic knapsack with finite capacity (i.e., $c_l < \infty$) by $P_{lp}[c_l; \epsilon_l; \sigma_l^2]$.

For the multi-rate network with multiple links we use the same notation as in Section 4.2 and make again the link independence assumption, so that class-q connections arrive to link l according to a Poisson process with the rate given by (4.7). By invoking the Pascal approximation, we obtain the probability that the capacity of link l is not available for a call of class p as

$$\hat{L}_{lp} = P_{lp}\left[c_l; \sum_{q \in \mathcal{P}_l} r_q \rho_q^v \prod_{\ell \in q, \ell \neq l} (1 - \hat{L}_{\ell q});\ \sum_{q \in \mathcal{P}_l} r_q^2 \rho_q^v \prod_{\ell \in q, \ell \neq l} (1 - L_{\ell q})\right], \quad q \in \mathcal{P}_l, \quad l \in \mathcal{L} \tag{4.16}$$

where the function $P_{lp}(\ \cdots\ ,\ \cdots\ ,\ \cdots\ )$ is given by (4.13) and we use $\hat{L}_{lp}$ to denote the approximation to the probability of blocking of voice path p on link l for the Pascal method; the same quantity for the knapsack method was denoted by L_{lp}. In general $\hat{L}_{lp} \neq L_{lp}$; however, in the important single-rate case ($r_p = 1$ for all $p \in \mathcal{P}$), we show in Appendix F that $\hat{L}_{lp} = L_{lp}$ ($p \in \mathcal{P}_l, l \in \mathcal{L}$).

As with the knapsack approximation of Section 4.2, there exists a nonunique solution to the fixed point equation (4.16). Once having found such a solution, B_p, the probability of blocking a class-p connection (or a voice call along path p) is approximated by (4.9); similarly, B_l, the probability of blocking any voice call on link l is approximated by (4.10).

The computational complexity and the accuracy of the Pascal approximation are very comparable with those of the knapsack approximation. Moreover, the Pascal approximation technique was shown in [6] to be asymptotically accurate (correct) under the same limiting regime as Kelly's and the knapsack approximations. Through our comparisons it has also been established that the Pascal approximation is accurate over a broader range of load parameters than Kelly's approximation.

4.4 Mitra's Approximation

The approximation technique suggested by Mitra [10] relies on a Taylor series expansion of the normalization constant of (3.10). Although it has the advantage that we can control its accuracy by increasing the number of terms in the Taylor series expansion, it has the disadvantage of converging only for a rather limited number of scenarios. Actually, we were not initially aware of that disadvantage, since [10] remains silent about it.

In [10], the approximation technique was applied to a tree network. However, in that paper there was not any stated constraint on the traffic loads and number of channels (of the circuits), without which convergence was unattainable. We simulated Mitra's results and found out that he only presented in his paper the scenarios for which convergence was attained. By increasing the loads we found out that the convergence was no longer guaranteed.

Consequently, although we were able to extend the applicability of Mitra's approximation method (a) to more general topologies than trees, (b) from single-rate networks to multi-rate networks, and (c) to the probability of data queueing (besides the probability of voice blocking), the severely limited range of convergence precludes the application of this method in most cases. Since all our simulation results using this method showed poor accuracy (except for the results presented in Mitra's paper about the tree network and for only the traffic loads reported there), we opted not to present the mathematical details of the application of this approximation method even for our novel work (i.e., the extensions to general topologies and to multi-rate networks).

4.5 Knapsack and Pascal Approximations for the Radio Network Model of Section 2.2

In Appendix G we show that for networks with single-rate traffic the knapsack and Pascal methods provide identical approximations to the probabilities of voice blocking and data queueing. Consequently for the radio model of Sections 2.1.2 and 2.2.2, which is characterized by $r_p = 1$ for all paths $p \in \mathcal{P}$, we use only the knapsack approximation in deriving the numerical results for the voice blocking probability in Section 12.

The application of the knapsack approximation method to the evaluation of the probability of voice blocking for the radio model of Sections 2.1.2 and 2.2.2 is a straightforward modification of the approach described in detail in Section 4.2. In particular, we replace l (links) by n (nodes), $\mathcal{L}$ by $\mathcal{N}$, $\mathcal{P}_l$ (set of paths using link l) by $\mathcal{P}_n$ (set of paths intersecting at node n), c_l (link capacities) by T_n (number of transceivers at nodes), and $r_p = 1$ (for all $p \in \mathcal{P}$). We are then able to use the results of subsection 4.3 directly.

5. KNAPSACK APPROXIMATION TO THE PROBABILITY OF DATA QUEUEING

In this section we describe in detail the application of the knapsack approximation to the probability of data queueing. This development appears for the first time in this report. The approximation is first developed for general multi-rate networks in Section 5.1 and then modified in Section 5.2 to be suitable for the multi-hop radio network of Sections 2.1.2 and 2.2.2.

5.1 General Multi-Hop Multi-Rate Networks

The key idea behind the application of the knapsack approximation to the probability of data queueing is to use the technique described in Section 4.2 to approximate the external sum in (3.13), while treating the internal sum (the sum with respect to n_l^d) as a single entity for given values of $(\underline{N}^v, \underline{N}^s)$ and use Proposition 3.1. This approach is described in the following in detail.

As for the case of voice traffic (Section 4.2) in order to derive the knapsack approximation to the probability of data queueing in link l, Q_l, we first consider the probability of queueing for a single-link multi-rate system. As we saw in the comments after Proposition 3.1 of Section 3, the probability of data queueing for link l can be expressed as in (3.13) which we may modify as

$$\begin{aligned} Q_l &= 1 - \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega_s} \left[\sum_{n_l^d=0}^{c_l'-1} I[(n_l^v, n_l^s) \neq (c_l, 0)] P(n_l^d | \underline{N}^v, \underline{N}^s) \right] P(\underline{N}^v, \underline{N}^s) \\ &= 1 - \sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l, 0)] \cdot \left[\sum_{n_l^d=0}^{c_l'-1} P(n_l^d | c_l') \right] P(k,m) \end{aligned} \tag{5.1}$$

where

$$\Omega(k,m) = \left\{ (\underline{N}^v, \underline{N}^s) \middle| \; 0 \leq n_p^s \leq n_p^v, \;\; 0 \leq r_p n_p^v \leq c_l, \;\; p \in \mathcal{P}_l; \sum_{p \in \mathcal{P}_l} r_p n_p^s = m \; ; \;\; \sum_{p \in \mathcal{P}_l} r_p n_p^v = k \right\} \tag{5.2}$$

while

$$P(k,m) = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k,m)} P(\underline{N}^v, \underline{N}^s) \tag{5.3}$$

with

$$\sum_{k=0}^{c_l} \sum_{m=0}^{k} P(k,m) = 1 \tag{5.4}$$

and

$$c_l' = c_l - k + m \tag{5.5}$$

is the number of channels available for data, where c_l is the total number of channels in the link, k is the number of voice calls in progress, m of which are presently in silence mode, and $P(n_l^d|\underline{N}^v, \underline{N}^s)$ is the steady-state probability of an $M/M/c_l'$ or an $M/D/c_l'$ system. In (5.1) we simplified the notation for $P(n_l^d|\underline{N}^v, \underline{N}^s)$ to $P(n_l^d|c_l')$ because it depends on $(\underline{N}^v, \underline{N}^s)$ only through c_l',

The parameters k and m in (5.1)-(5.5) represent the total bandwidth (channel capacity) occupied by the number of total ongoing voice calls and of voice calls in silent mode, respectively, on which the expression for queueing probability under the M/D/c data model is conditioned. The importance of this conditioning should not be underestimated; the fact that we can obtain efficient recursive expressions for the probability distribution (mass function) of (k,m) (see Appendix C) enables us to perform the final averaging with respect to k and m and evaluate the unconditional expressions for the performance measures of interest.

Regarding the evaluation of

$$\bar{P}(c_l', \rho_l^d) = \sum_{n_l^d=0}^{c_l'-1} P(n_l^d|c_l') \tag{5.6}$$

which is a factor in Eq. (5.1), we proceed as follows depending on the model used for the data traffic

(a) For an $M/M/c_l'$ data model, $\bar{P}(c_l', \rho_l^d)$ is given by a well known formula [1] (also see $1 - \Pi_w$ in Appendix B.1), which can be put in the recursive form

$$\bar{P}(c_l', \rho_l^d) = \frac{1}{1 + \left(\frac{c_l' - \rho_l^d}{\rho_l^d}\right) \cdot \frac{c_l' - 1 - \rho_l^d \cdot \bar{P}(c_l' - 1, \rho_l^d)}{(c_l' - 1 - \rho_l^d) \cdot \bar{P}(c_l' - 1, \rho_l^d)}} \tag{5.7a}$$

$$\bar{P}(0, \rho_l^d) = 1 \tag{5.7b}$$

Consequently, $\bar{P}(c_l', \rho_l^d)$ can be evaluated iteratively.

(b) For an $M/D/c_l'$ data model, rather than evaluating $\bar{P}(c_l', \rho^d)$ directly, we evaluate $P(n_l^d | c_l')$ iteratively via a special-purpose overrelaxation method (see Tijms [13] and Appendix B), and sum up the resulting $P(n_l^d | c_l')$ to obtain $\bar{P}(c_l', \rho_l^d)$.

The quantity in (5.6) represents the total probability that the residual data capacity c_l' of the l-th link [given by (5.5), which depends on the number of active voice calls] is not exceeded by the number of data messages using link l. Thus, it corresponds to the conditional probability of no queueing of data (or alternatively one minus the probability of finding the system totally occupied) when conditioned on the voice state.

In order to evaluate $P(k, m)$ we define

$$P'(k, m) = P(k, m)/P(0, 0) \tag{5.8a}$$

as the normalized version of $P(k, m)$. This results in

$$\left[\sum_{k=0}^{c_l} \sum_{m=0}^{k} P'(k, m) \right]^{-1} = P(0, 0). \tag{5.8b}$$

Then $P'(k, m)$ can be obtained iteratively from Proposition 5.1 that follows, and the probability of data queueing Q_l for link l (and a single-link network) can be approximated by

$$\begin{aligned} Q_l(c_l; \rho_p^v, p \in \mathcal{P}_l) &= 1 - \sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k, m) \neq (c_l, 0)] \cdot \bar{P}(c_l', \rho_l^d) P(k, m) \\ &= 1 - \frac{\sum_{k=0}^{c_l} \sum_{m=0}^{k} I(c_l' \neq 0) \cdot \bar{P}(c_l', \rho_l^d) \cdot P'(k, m)}{\sum_{k=0}^{c_l} \sum_{m=0}^{k} P'(k, m)}. \end{aligned} \tag{5.9}$$

In (5.9) we used the notation $Q_l(c_l; \rho_p^v, p \in \mathcal{P}_l)$ in order to emphasize the dependence of the probability of data queueing on the voice loads $(\rho_p^v, p \in \mathcal{P}_l)$. The importance of this notation will become clear below when the approximation for a multi-link network is obtained.

Proposition 5.1

$P'(k,m)$ satisfies

$$P'(k,m) = \begin{cases} \frac{1}{m}\frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v P'(k-r_p, m-r_p)\ , & \text{if } r_p \le m \le k \le c_l \\ \frac{1}{k-m}\frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v P'(k-r_p, m)\ , & \text{if } 0 \le m < r_p \le k \le c_l \\ 1\ , & \text{if } k=0, m=0 \\ 0\ , & \text{if } k \text{ or } m \text{ are not positive integer multiples of } r_p \end{cases} \tag{5.10a}$$

$$P'(0,0) = 1 \tag{5.10b}$$

$$P'(k,m) = 0 \text{ if } \begin{cases} m > k, \text{ or } k < 0, \text{ or } m < 0, \\ \text{or if } (k,m) \text{ can not be represented as the linear combination of } r_p,\ p \in \mathcal{P}_l \end{cases} \tag{5.10c}$$

Proof: It is provided in Appendix C.

Finally, to evaluate the probability of data queueing for a multi-link network we proceed as follows. We use the thinned voice load (under the interlink independence assumption)

$$[\rho_p^v]' = \rho_p^v \prod_{\ell\in p,\ell\neq l} (1 - L_{\ell p}) \tag{5.11}$$

offered to link l from voice path p in place of the original ρ_p^v in the functional form of (5.9). The resulting approximation to the probability of queueing at link l (and a multi-link network) is

$$Q_l\left(c_l; \rho_p^v \prod_{\ell\in p,\ell\neq l} (1 - L_{\ell p}), p \in \mathcal{P}_l\right) \tag{5.12}$$

where $L_{\ell p}$ is the probability of blocking voice calls taking path p at link ℓ, which is obtained via the knapsack approximation on the voice part (Section 4.2).

A comment is in order here regarding the theoretical accuracy of the approximation; the practical accuracy is very satisfactory as demonstrated in Section 11 where the various approximations are compared with the results of accurate Monte-Carlo summation. Assuming that the computation of the data portion $\bar{P}(c_l, \rho_l^d)$ of (5.1) is accurate, we claim that the same limiting regime (i.e., when both voice load and number of channels in each link are large but their ratio is held fixed) that yields the probability of voice blocking B_l asymptotically accurate also yields the probability of data queueing Q_l asymptotically accurate. Our justification is at this point only a conjecture (it has not been proved mathematically) and is based on the facts that (a) the same inter-link voice independence assumption was used as in the voice blocking probability calculation, and (b) in the evaluation of Q_l, the knapsack approximation was applied to the voice portion (i.e., to determine the expected residual data capacity) for which it has already been shown to be accurate. We anticipate to be able to establish this claim rigorously in the future.

5.2 Radio Network Model of Sections 2.1.2, 2.2.2, and 2.4.2

As discussed in Sections 2.1.2 and 2.2.2 the radio network model considered in this report assumes that the data traffic over link $l = (n_1, n_2) \in \mathcal{L}$ connecting the network nodes $n_1, n_2 \in \mathcal{N}$ follows an $M/D/c_l'$ system model independently from the other links of the network. The residual capacity available for data is given by

$$c_l' = \min\left\{T_{n_1} - \sum_{p\in\mathcal{P}_{n_1}} (n_p^v - n_p^s), T_{n_2} - \sum_{p\in\mathcal{P}_{n_2}} (n_p^v - n_p^s)\right\},\ l \in \mathcal{L},\ n_1, n_2 \in \underline{N} \tag{5.13}$$

where

$$\sum_{p\in\mathcal{P}_n} n_p^v \leq T_n \quad n \in \mathcal{N}.$$

It is assumed that the data are queued at buffers (of infinite capacity) available at the node transceivers. The above equation for the data link capacity expresses the dependence of the M/D/c data model on the current voice traffic at nodes n and m. However, it does not address the way in which a node's unused transceivers are allocated to support data links with each of its neighbors, as was discussed in Section 2.4.2.

To simplify the notation we consider the link $l = (n_1, n_2) = (1, 2)$ connecting the network nodes $1, 2 \in \mathcal{N}$. The above data capacity can be written as

$$c_l' = \min\{T_1 - k_1 + m_1,\ \ T_2 - k_2 + m_2\} \tag{5.14}$$

where

$$0 \leq k_i = \sum_{p\in\mathcal{P}_i} n_p^v \leq T_i, \qquad i = 1, 2 \tag{5.15a}$$

$$0 \le m_i = \sum_{p \in \mathcal{P}_i} n_p^s \le k_i, \qquad i = 1, 2 \tag{5.15b}$$

and $\mathcal{P}_i$ is the set of paths containing node i, for $i = 1, 2$.

To employ the knapsack approximation in this case for a single-link network involves defining

$$P(k_1, m_1, k_2, m_2) = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k_1, m_1, k_2, m_2)} P(\underline{N}^v, \underline{N}^s) \tag{5.16}$$

where

$$\Omega(k_1, m_1, k_2, m_2) = \left\{ (\underline{N}^v, \underline{N}^s) \mid 0 \le n_p^s \le n_p^v,\ p \in \mathcal{P};\ \sum_{p \in \mathcal{P}_i} n_p^v = k_i,\ \sum_{p \in \mathcal{P}_i} n_p^s = m_i,\ \ i = 1, 2 \right\} \tag{5.17}$$

$$\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2$$

$$\sum_{k_1=0}^{T_1} \sum_{m_1=0}^{k_1} \sum_{k_2=0}^{T_2} \sum_{m_2=0}^{k_2} P(k_1, m_1, k_2, m_2) = 1$$

and writing

$$Q_l = 1 - \sum_{k_1=0}^{T_1} \sum_{m_1=0}^{k_1} \sum_{k_2=0}^{T_2} \sum_{m_2=0}^{k_2} I(c_l' \ne 0) \cdot \bar{P}(c_l', \rho_l^d) \cdot P(k_1, m_1, k_2, m_2) \tag{5.18}$$

where

$$\bar{P}(c_l', \rho_l^d) = \sum_{n_l^d=0}^{c_l'-1} P(n_l^d | c_l')$$

is provided by the M/D/c data model (Appendix B).

After the appropriate normalization

$$P'(k_1, m_1, k_2, m_2) = P(k_1, m_1, k_2, m_2)/P(0,0,0,0) \tag{5.19}$$

and using the approach of Section 5.1, the above single-link approximation is put in the form

$$Q_l(T_1, T_2; \rho_p^v, p \in \mathcal{P}_1 \cup \mathcal{P}_2) = 1 - \frac{\sum_{k_1=0}^{T_1} \sum_{m_1=0}^{k_1} \sum_{k_2=0}^{T_2} \sum_{m_2=0}^{k_2} I(c_l' \ne 0) \bar{P}(c_l', \rho_l^d) P'(k_1, m_1, k_2, m_2)}{\sum_{k_1=0}^{T_1} \sum_{m_1=0}^{k_1} \sum_{k_2=0}^{T_2} \sum_{m_2=0}^{k_2} P'(k_1, m_1, k_2, m_2)} \tag{5.20}$$

with

$$c_l' = \min\{T_1 - k_1 + m_1,\ \ T_2 - k_2 + m_2\}\,. \tag{5.14}$$

First we consider the case $\mathcal{P}_1 = \mathcal{P}_2$, which means that the same set of paths passes through node 1 and node 2. In this case, $k_1 = k_2$, $m_1 = m_2$, and Q_l simplifies to

$$Q_l = 1 - \frac{\sum_{k=0}^{T}\sum_{m=0}^{k} I(c_l' \neq 0)\cdot \bar{P}(c_l', \rho_l^d)\cdot P'(k,m)}{\sum_{k=0}^{T}\sum_{m=0}^{k} P'(k,m)} \tag{5.21a}$$

where

$$T = \min\{T_1, T_2\} \tag{5.21b}$$

$$c_l' = \min\{T_1, T_2\} - k + m = T - k + m \tag{5.21c}$$

The recursion for $P'(k,m)$ is given by (5.10).

Next we consider the case $\mathcal{P}_1 \neq \mathcal{P}_2$, in which case calls of one or more types are supported by only one of the nodes of interest. The following Proposition holds

Proposition 5.2

$P'(k_1, m_1, k_2, m_2)$ satisfies the recursion

$$P'(k_1, m_1, k_2, m_2) =$$

$$= \begin{cases} \frac{1}{m_1}\cdot\frac{\alpha}{\alpha+\beta}\left[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c}\rho_p^v P'(k_1-1, m_1-1; k_2, m_2) + \sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2}\rho_p^v P'(k_1-1, m_1-1; k_2-1, m_2-1)\right]; \\ \qquad\qquad \text{if } \ 1 \le m_1 \le k_1 \le T_1, 1 \le m_2 \le k_2 \le T_2; \\ \frac{1}{m_1}\cdot\frac{\alpha}{\alpha+\beta}\left[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c} r_p^{(1)}\rho_p^v P'(k_1-1, m_1-1; k_2, m_2)\right]; \\ \qquad\qquad \text{if } \ 1 \le m_1 \le k_1 \le T_1,\ m_2 = 0 \le k_2 \le T_2; \\ \frac{1}{m_2}\cdot\frac{\alpha}{\alpha+\beta}\left[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c}\rho_p^v P'(k_1, m_1; k_2-1, m_2-1)\right]; \\ \qquad\qquad \text{if } \ m_1 = 0 \le k_1 \le T_1,\ 1 \le m_2 \le k_2 \le T_2 \\ \frac{1}{k_1-m_1}\cdot\frac{\beta}{\alpha+\beta}\left[\sum_{p\in\mathcal{P}_2\cap\mathcal{P}_1^c}\rho_p^v P'(k_1-1, m_1; k_2, m_2) + \sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2}\rho_p^v P'(k_1-1, m_1; k_2-1, m_2)\right]; \\ \qquad\qquad \text{if } \ m_1 = 0 < k_1 \le T_1, m_2 = 0 < k_2 \le T_2 \\ 1\,; \qquad\qquad \text{if } \ k_1 = 0,\ m_1 = 0,\ k_2 = 0,\ m_2 = 0 \\ 0\,; \qquad\qquad \text{otherwise} \end{cases} \tag{5.22}$$

where $\mathcal{P}_2^c$ denotes the complement of the set $\mathcal{P}_2$.

Proof: Refer to Appendix D where a more general proof involving constraints of the form

$$0 \leq k_i = \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^v \leq Z_i, \qquad i = 1, 2$$

$$0 \leq m_i = \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^s \leq k_i, \qquad i = 1, 2$$

is provided. For the approximation described in this section we considered only the special case

$$r_p^{(1)} = r_p^{(2)} = 1 \quad p \in \mathcal{P}_1 \cup \mathcal{P}_2$$

and

$$Z_i = T_i, \quad i = 1, 2$$

For the multi-link network, the knapsack approximation for node $l = (n_1, n_2)$ is obtained with the help of the thinned load method, thus we use the function of (5.20) (where $1 = n_1$ and $2 = n_2$) with voice loads $\rho_p^v \prod_{n \in p, n \neq n_1, n_2} (1 - L_{np})$ instead of ρ_p^v to get

$$Q_l \left(T_{n_1}, T_{n_2} \; ; \; \rho_p^v \prod_{n \in p, n \neq n_1, n_2} (1 - L_{np}), p \in \mathcal{P}_1 \cup \mathcal{P}_2 \right), \quad l = \{n_1, n_2\} \in \mathcal{L} \tag{5.23}$$

for a fixed value of ρ_l^d and

$$c_l' = \min \{ T_{n_1} - k_1 + m_1, T_{n_2} - k_2 + m_2 \}.$$

6. PASCAL APPROXIMATION TO THE PROBABILITY OF DATA QUEUEING

In this section we describe in detail the application of the Pascal approximation to the evaluation of the probability of data queueing, a development that appears in this report for the first time. This approximation is first developed for general multi-rate networks in Section 6.1; in Section 6.2, we describe how it can be modified to suit the multi-hop radio network of Sections 2.1.2 and 2.2.2.

6.1 General Multi-Rate Networks

In Section 4.3 the Pascal approximation used a one dimensional birth-death process whose equilibrium probability mass function is the Pascal distribution to approximate the number of busy circuits (paths with ongoing voice calls). In order to approximate the probability of data queueing given by (3.13), which involves conditioning on the total number of voice calls and the on the number of silent (inactive) calls, we need to extend the Pascal approximation to two dimensions.

In this context, we consider the two-dimensional birth-death process $(k-m,m)$ (where $k-m$ roughly represents the total rate used by all active calls and m the rate of all inactive calls) on the state space $\{(k,m) \mid 0 \le k \le c, 0 \le m \le k\}$ with the transition diagram of Figure 1, where

$$\lambda_{j,i} = \left[\frac{\epsilon_j^2}{\sigma_j^2} + i\left(1 - \frac{\epsilon_j}{\sigma_j^2}\right)\right], \qquad j = 1,2,\ i = 0,1,2,\ldots,c \tag{6.1}$$

and ϵ_j^2, σ_j^2, for $j = 1,2$, are given positive numbers. Let $q(k-m,m)$ denote the equilibrium probability of being in state $(k-m,m)$. Furthermore, consider the associated process (k,m) and denote by $\hat{P}(k,m)$ the equilibrium probability of being at state (k,m).

From the local balance equations for the birth-death process $(k-m,m)$ we know that $q(k-m,m)$ must satisfy the conditions

$$(k-m)\cdot q(k-m,m) = \lambda_{1,k-1-m}\cdot q(k-1-m,m) \tag{6.2}$$

and

$$m\cdot q(k-m,m) = \lambda_{2,m-1}\cdot q(k-m,m-1). \tag{6.3}$$

The resulting conditions for $\hat{P}(k,m)$ are

$$(k-m)\cdot \hat{P}(k,m) = \lambda_{1,k-1-m}\cdot \hat{P}(k-1,m) \tag{6.4}$$

and

$$m \cdot \hat{P}(k,m) = \lambda_{2,m-1} \cdot \hat{P}(k-1,m-1). \tag{6.5}$$

Using the above equations we can evaluate $\hat{P}(k,m)$ from the following recursion

$$\hat{P}(k,m) = \begin{cases} \frac{\lambda_{1,k-1}}{k} \cdot \hat{P}(k-1,0), & m = 0;\ 1 \le k \le c \\ \frac{\lambda_{2,m-1}}{m} \cdot \hat{P}(k-1,m-1), & 1 \le m \le k;\ 1 \le k \le c \end{cases} \tag{6.7a}$$

where

$$\sum_{k=0}^{c} \sum_{m=0}^{k} \hat{P}(k,m) = 1. \tag{6.8a}$$

If instead of $\hat{P}(k,m)$ we use the normalized version

$$\hat{P}'(k,m) = \hat{P}(k,m)/\hat{P}(0,0)$$

then the recursion (6.7a)-(6.8a) becomes

$$\hat{P}'(k,m) = \begin{cases} \frac{\lambda_{1,k-1}}{k} \cdot \hat{P}'(k-1,0), & m = 0;\ 1 \le k \le c \\ \frac{\lambda_{2,m-1}}{m} \cdot \hat{P}'(k-1,m-1), & 1 \le m \le k;\ 1 \le k \le c \end{cases} \tag{6.7b}$$

with initial condition

$$\hat{P}'(0,0) = 1 \tag{6.8b}$$

which is easier to evaluate [it does not require knowledge of $P(0,0)$] and results in

$$\left[\sum_{k=0}^{c} \sum_{m=0}^{k} \hat{P}'(k,m) \right]^{-1} = \hat{P}(0,0).$$

Proposition 6.1

When $c = \infty$, $\hat{P}(k,m)$ is two-dimensional Pascal-distributed with the following parameters:

$$E[k-m] = \epsilon_1 \tag{6.9}$$

$$E[m] = \epsilon_2 \tag{6.10}$$

$$var(k-m) = \sigma_1^2 \tag{6.11}$$

$$var(m) = \sigma_2^2 \tag{6.12}$$

$$cov(k - m, m) = 0. \tag{6.13}$$

Proof: It is provided in Appendix D.

The application of the Pascal distribution to the probability of data queueing involves (as in the case of the knapsack approximation) considering first a single-link multi-rate network, applying the Pascal approximation, and then obtaining the result for the multi-link case using thinned voice loads in place of the initial voice loads. We start with the following proposition

Proposition 6.2

For the singie-link multi-rate network, when the link capacity $c = \infty$, denote by n_v the number of channels occupied by voice calls in the link and denote by n_s the number of channels occupied by silence voice calls in the link, then

$$E[n_v] = \sum_{p \in \mathcal{P}} r_p \rho_p^v \tag{6.14}$$

$$E[n_s] = \frac{\alpha}{\alpha + \beta} \cdot \sum_{p \in \mathcal{P}} r_p \rho_p^v \tag{6.15}$$

$$E[n_v - n_s] = \frac{\beta}{\alpha + \beta} \cdot \sum_{p \in \mathcal{P}} r_p \rho_p^v \tag{6.16}$$

$$var(n_v) = \sum_{p \in \mathcal{P}} r_p^2 \rho_p^v \tag{6.17}$$

$$var(n_s) = \frac{\alpha}{\alpha + \beta} \sum_{p \in \mathcal{P}} r_p^2 \rho_p^v \tag{6.18}$$

$$var(n_v - n_s) = \frac{\beta}{\alpha + \beta} \sum_{p \in \mathcal{P}} r_p^2 \rho_p^v \tag{6.19}$$

$$cov(n_v - n_s, n_s) = 0. \tag{6.20}$$

Proof: It is provided in Appendix E.

From the above two propositions we observe that the infinite capacity single-link multi-rate model employs the same means and variances for $(n_v - n_s)$ and n_s as the two-dimensional birth-death process with parameters $\epsilon_1 = E[n_v - n_s]$, $\epsilon_2 = E[n_s]$, $\sigma_1^2 =$

$var(n_v - n_s)$, and $\sigma_2^2 = var(n_s)$. Note that the covariances $cov\{n_v - n_s, n_s\}$ and $cov\{k - m, m\}$ are both zero. Therefore, as in the voice case, the probability of data queueing in (3.13) for the finite-capacity single-link multi-rate model can approximated by

$$\hat{Q}_l\left(c_l; \rho_p^v, p \in \mathcal{P}_l\right) = 1 - \sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l, 0)] \cdot \bar{P}(c_l', \rho_l^d) \cdot \hat{P}(k,m)$$

$$= 1 - \frac{\sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l, 0)] \cdot \bar{P}(c_l', \rho_l^d) \cdot \hat{P}'(k,m)}{\sum_{k=0}^{c_l} \sum_{m=0}^{k} \hat{P}'(k,m)} \tag{6.21}$$

where $\bar{P}(c_l', \rho_l^d)$ is given by (5.7a)-(5.7b) for an M/M/c_l' data model and can be evaluated from Appendix B for an M/D/c_l' data model, and $\hat{P}'(k,m)$ is the equilibrium probability of the above birth-death process with parameters

$$\epsilon_1 = \frac{\beta}{\alpha+\beta} \sum_{p \in \mathcal{P}_l} r_p \rho_p^v \tag{6.22}$$

$$\epsilon_2 = \frac{\alpha}{\alpha+\beta} \sum_{p \in \mathcal{P}_l} r_p \rho_p^v \tag{6.23}$$

$$\sigma_1^2 = \frac{\beta}{\alpha+\beta} \sum_{p \in \mathcal{P}_l} r_p^2 \rho_p^v \tag{6.24}$$

$$\sigma_2^2 = \frac{\alpha}{\alpha+\beta} \sum_{p \in \mathcal{P}_l} r_p^2 \rho_p^v \tag{6.25}$$

which can be evaluated iteratively via (6.7b)-(6.8b). We used the notation $\hat{Q}_l\left(c_l; \rho_p^v, p \in \mathcal{P}_l\right)$ in order to emphasize the functional dependence of the probability of data queueing on the voice loads $(\rho_p^v, p \in \mathcal{P}_l)$. This is necessary for the extension to the multi-link case that follows.

For the multi-link multi-rate network we use as the approximation to the probability of data queueing at link l the expression

$$\hat{Q}_l\left(c_l; \rho_p^v \prod_{\ell \in p, \ell \neq l} (1 - \hat{L}_{\ell p}), p \in \mathcal{P}_l\right) \tag{6.26}$$

where $\hat{L}_{\ell p}$ is the (approximate) probability of voice call taking path p being blocked at link ℓ, which is obtained via the Pascal approximation as in Section 4.3. As it was done in

Section 5, we again used the interlink independence approximation for voice and thinned the voice loads ρ_p^v to $\rho_p^v \prod_{\ell \in p, \ell \neq l}(1 - \hat{L}_{\ell_p})$ for $p \in \mathcal{P}_l$ in (6.26).

The same comments about the practical accuracy and the asymptotic accuracy of this approximation with those made at the end of Section 5 are valid here.

6.2 Radio Network Model of Sections 2.1.2, 2.2.2, and 2.4.2

The general approach described in Section 6.1 above can be extended to derive a four-dimensional Pascal approximation for the approximation of probability of queueing data of the radio network model of Sections 2.1.2 and 2.2.2. The basic steps are similar to those detailed in Section 5.2 for the knapsack approximation. The key step is again the derivation of recursive expressions for the quantity $\hat{P}(k_1, m_1, k_2, m_2)$ defined as in (5.15) but evaluated via the Pascal method for the multi-rate case. We omit the details here which are similar to those provided in Appendix D. For the single-rate case characterizing the radio network model of this report the knapsack and Pascal approximations yield identical results (see Appendix F) and thus there is no need for additional work.

7. EXTENSION OF THE APPROXIMATIONS TO M/D/c DATA MODELS AND THE AVERAGE QUEUEING DELAY AS THE PERFORMANCE MEASURE

As discussed in Section 2 the performance measures of interest for the data traffic are

$Q_l \triangleq$ probability of data queued at link l ($l \in \mathcal{L}$),

$W_l \triangleq$ average waiting time (not including service time) of data in queue at link l ($l \in \mathcal{L}$),

and their averages $\bar{Q}$ and $\bar{W}$ [refer to (2.1b)-(2.1c)] with respect to the data loads of the links. In this section we first show (Section 7.1) that the approximations to the probability of data queueing of Sections 5 and 6 are applicable not only to the M/M/c data model but also to the M/D/c data model. By contrast, for the average data queueing delay these approximations are shown (Section 7.2) to apply only to the M/D/c data model. Finally, in Section 7.3 a modified network model that guarantees finite data delays by dedicating a portion of the link capacity (or the number of node transceivers) to exclusive data use is provided.

7.1 Approximations to the Probability of Data Queueing for M/D/c Data Models

Recall that for the **M/M/c data model** and a multi-rate system, the **probability of data queueing** Q_l takes the form

$$Q_l = Pr(n_l^d \geq c_l - k_l^v + k_l^s) = 1 - Pr\left[c_l - k_l^v + k_l^s > n_l^d, \text{ where } (k_l^v, k_l^s) \neq (c_l, 0)\right]$$

$$= 1 - \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega_s} \left[\sum_{n_l^d=0}^{c_l'-1} P(n_l^d | c_l')\right] P(\underline{N}^v, \underline{N}^s) \tag{7.1}$$

where

$$k_l^v = \sum_{p \in \mathcal{P}_l} r_p n_p^v \tag{7.2}$$

$$k_l^s = \sum_{p \in \mathcal{P}_l} r_p n_p^s \tag{7.3}$$

and

$$c_l' = c_l - k_l^v + k_l^s. \tag{7.4}$$

$P(\underline{N}^v, \underline{N}^s)$ is the steady state probability of the voice calls evaluated in Section 3, and $P(n_l^d|c_l')$ is the steady state probability of the number of data packets in an $M/M/c_l'$ system. The application of the knapsack and Pascal methods for obtaining approximations to the probability of data queueing Q_l was described in Sections 5 and 6, respectively, for an M/M/c data model.

The corresponding formula for the **M/D/c data model** is obtained by simply replacing the $P(n_l^d|c_l')$ of an M/M/c_l' system by that of an M/D/c_l' system (see Appendix B). Consequently, the application of the knapsack and Pascal approximation methods to Q_l and an M/D/c data model is a straightforward extension of the results of Sections 5 and 6.

7.2 Approximations to the Average Queueing Delay of Data

The **average data queueing delay** W_l for the **M/M/c data model** and the **M/D/c data model** takes (upon application of Little's formula) the form

$$W_l = \frac{\text{average number of data packets in the queue at link } l}{\text{average data rate to link } l} = \frac{N_l^Q}{F_l^d} \tag{7.5}$$

where

$$N_l^Q = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega_s} \left[\sum_{n=0}^{\infty} n P(n_l^d = c_l' + n | c_l') \right] P(\underline{N}^v, \underline{N}^s) \tag{7.6}$$

$$F_l^d = \rho_l^d \cdot \mu_l^d \tag{7.7}$$

and $P(n_l^d|c_l')$ and $P(\underline{N}^v, \underline{N}^s)$ are as described above for the two models. Therefore, W_l can be evaluated by applying the various approximations on N_l^Q and treating $\sum_{n=0}^{\infty} nP(n_l^d = c_l' + n \mid c_l')$ in the same manner as $\sum_{n_l^d=0}^{c_l'-1} P(n_l^d \mid c_l')$ was treated in the original performance measure, i.e., in the probability of data queueing. Finally, $\sum_{n=0}^{\infty} nP(n_l^d = c_l' + n \mid c_l')$ can be evaluated in closed form (omitted here) for an M/M/c_l' system and via Tijms' algorithm (see [13] and Appendix B) for an M/D/c_l' system.

The application of the approximation methods of Sections 5 and 6 to the queueing delay for data W_l and a M/D/c model is described in more detail next.

7.2.1 Knapsack Approximation

As discussed above we apply the approximation methods directly to the evaluation of the average number of data packets in the queue (N_l^Q) required for the evaluation of the average queueing delay $W_l = N_l^Q / F_l^d$. As in Section 5 we start with a single-link multiple-rate scenario and write for link l

$$N_l^Q = \sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l,0)] \cdot \left[\sum_{n=0}^{\infty} nP(n_l^d = c_l' + n \mid c_l') \right] \cdot P(k,m) \tag{7.8}$$

where

$$c_l' = c_l - k + m \tag{7.9}$$

$$P(k,m) = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k,m)} P(\underline{N}^v, \underline{N}^s) \tag{7.10}$$

and

$$\Omega(k,m) = \left\{ (\underline{N}^v, \underline{N}^s) \mid 0 \leq n_p^s \leq n_p^v;\ 0 \leq r_p n_p^v \leq c_p;\ p \in \mathcal{P}_l, \sum_{p \in \mathcal{P}_l} r_p n_p^s = m; \sum_{p \in \mathcal{P}_l} r_p n_p^v = k \right\}. \tag{7.11}$$

Let

$$\bar{N}(c_l', \rho_l^d) = \sum_{n=0}^{\infty} nP(n_l^d = c_l' + n \mid c_l') \tag{7.12}$$

which is the average number of data packets in the queue for an M/D/c_l' system, and is isolated from other terms in N_l^Q and can thus be evaluated separately. The evaluation of $\bar{N}(c_l', \rho_l^d)$ is straightforward from Tijms' algorithm (see [13]) after the steady-state probability for M/D/c_l' data has been obtained.

Then, following Section 5 we obtain an approximation to N_l^Q as

$$N_l^Q \left(c_l;\ \rho_p^v \prod_{\ell \in \mathcal{P}_l, \ell \neq l} (1 - L_{\ell p}), p \in \mathcal{P}_l \right) \tag{7.13}$$

where $L_{\ell p}$ is the approximate voice blocking probability for link ℓ across voice path p evaluated in Section 4.2 (via the knapsack approximation) and $N_l^Q(c_l;\ \rho_p^v,\ p \in \mathcal{P}_l)$ is a function of c_l and $(\rho_p^v,\ p \in \mathcal{P}_l)$ given by

$$N_l^Q[c_l;\ \rho_p^v,\ p \in \mathcal{P}_l] = \frac{\sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l,0)] \cdot \bar{N}(c_l', \rho_l^d) \cdot P'(k,m)}{\sum_{k=0}^{c_l} \sum_{m=0}^{k} P'(k,m)} \tag{7.14}$$

and $P'(k,m)$ is given by the same equations (5.10a)-(5.10c) of Section 5.

7.2.2 Pascal Approximation

For the Pascal approximation, we define $\bar{N}(c_l', \rho_l^d)$ as in (7.12) and still use the expressions (7.8)-(7.11) as in the knapsack method above. However, for the average length of data queue of link l (and a single-link network) we now use

$$\hat{N}_l^Q[c_l;\ \rho_p^v,\ p \in \mathcal{P}_l] = \frac{\sum_{k=0}^{c_l} \sum_{m=0}^{k} I[(k,m) \neq (c_l, 0)] \cdot \bar{N}(c_l', \rho_l^d) \cdot \hat{P}'(k,m)}{\sum_{k=0}^{c_l} \sum_{m=0}^{k} \hat{P}'(k,m)} \tag{7.15}$$

instead of (7.14), where $\hat{P}'(k,m)$ is now obtained from (6.7b) as described in Section 6. Subsequently, the approximation to N_l^Q (for a multi-link network) is obtained again as

$$\hat{N}_l^Q \left(c_l;\ \rho_p^v \prod_{\ell \in \mathcal{P}_l, \ell \neq l} (1 - \hat{L}_{\ell p}), p \in \mathcal{P}_l \right) \tag{7.16}$$

where $\hat{L}_{\ell p}$ is the approximate voice blocking probability for link ℓ across voice path p evaluated in Section 4.3 (via the Pascal approximation). Finally, the desired performance measure W_l is obtained as $W_l = \frac{\hat{N}_l^Q}{F_l^d}$.

7.3 Network Model Modification for Guaranteeing Finite Data Delays

Since the link capacity available for data given by (7.4) can take the value 0 we can not guarantee finite data delays unless we assume that a (small) fraction of the link capacity is always reserved for data use.

For the **multi-rate network model** of Sections 2.1.1 and 2.2.1 the above requirement implies that

$$c_l' = c_l^d + c_l^v - \sum_{p \in \mathcal{P}_l} r_p n_p^v + \sum_{p \in \mathcal{P}_l} r_p n_p^s \tag{7.17}$$

where c_l^d and c_l^v are the portions of the link capacity reserved for data and voice traffic respectively; if c_l^v is not used by the current voice traffic it can be used by the data traffic as (7.17) indicates.

For the **single-rate radio network model** of Sections 2.1.2, 2.2.2, and 2.4.2 the above requirement implies that the capacity available for data at link $l = (n_1, n_2)$ is

$$c_l' = \min\left\{T_{n_1}^d, T_{n_2}^d\right\} + \min\left\{T_{n_1}^v - \sum_{p \in \mathcal{P}_{n_1}} (n_p^v - n_p^s), T_{n_2}^v - \sum_{p \in \mathcal{P}_{n_2}} (n_p^v - n_p^s)\right\}, l \in \mathcal{L},\ n_1, n_2 \in \mathcal{N} \tag{7.18}$$

where T_n^d represents the number of transceivers at node n that are dedicated to data and T_n^v represents the number of transceivers which are primarily used by voice; the portion of T_n^v not occupied by voice calls can by used by data but voice maintains preemptive priority over data over this portion. The total number of transceivers at node n is

$$T_n = T_n^d + T_n^v, \quad n \in \mathcal{N}. \tag{7.19}$$

Similarly to the comment following eq. (5.13) and the discussion in Section 2.4.2, the above equation (7.18) does not address the way in which a node's unused transceivers are allocated to support data links with each of its neighbors.

Finally, note that, even if a portion of the link capacity (c_l^d or T_n^d) is set aside for exclusive data use, the data delay remains finite only for average data loads smaller than c_l^d (i.e., $\rho_l^d < c_l^d$); refer to the relevant comment (Comment 2) following Proposition 3.1 in Section 3.

8. EVALUATION OF PERFORMANCE MEASURES VIA THE MONTE-CARLO SUMMATION METHOD

For the purpose of comparison and testing the accuracy of the the knapsack and Pascal approximations to the performance measures of interest we also evaluate the probability of voice blocking, the probability of data queueing, and the queuing data delay via the Monte-Carlo summation method. In the next subsection this method is described in some detail and then in the following subsections it is explained how it is applied to the various performance measures.

The key difference between the approximation method described in this section and the approximation methods of the previous sections (Sections 4, 5, 6, and 7) is that the Monte-Carlo summation method can provide an estimate (approximation) to the performance measure of interest within any desired confidence interval at the cost of an increase in the number of calls to random number generators performed. In this sense the Monte-Carlo summation method constitutes our baseline for the true values of the performance measures of interest in this report, and, where exact values are not available, the accuracy of all other approximations is compared to the values generated by the Monte-Carlo simulation of the appropriate sums.

It should be emphasized that in contrast to the familiar Monte-Carlo simulation method the Monte-Carlo summation approach does not involve a simulation of the dynamic behavior of the system under study. As is explained in the following section, quantities related to system performance are evaluated by using known properties of their distributions in conjunction with a random generation process.

8.1 The Monte-Carlo Summation Method

It is now generally accepted that, in the absence of a special structure, multi-dimensional integration (or summation) is best performed by Monte Carlo methods [7], [15]. This method has been shown to be efficient and accurate in several applications involving expressions of the form

$$G = \sum_{\underline{n} \in \Omega} \prod_{k=1}^{K} q_k(n_k) \tag{8.1}$$

where Ω denotes the state space of an underlying K-dimensional vector stochastic process $\underline{n} = (n_1, \ldots, n_K)$ and $q_k(\cdot)$, $k = 1, \ldots, K$, are known functions. For the cases of interest to our study, G represents the normalization constant of (3.10) for product-form stochastic networks with voice traffic. However, as we have seen in Section 3 [eqs. (3.12a)-(3.12b) and (3.13)] and Section 7 [eqs. (7.5)-(7.7)], the performance measures of interest can actually be expressed as nonlinear functions of normalization constants. Therefore, expressions of the form

$$\Phi = \frac{\sum_{\underline{n} \in \Lambda} f(\underline{n}) q(\underline{n})}{\sum_{\underline{n} \in \Lambda} q(\underline{n})} \tag{8.2}$$

where $f(\cdot)$ is a known function are even more necessary than expressions of the form (8.1) in the evaluation procedure. The method for the evaluation of (8.1) is also applicable to (8.2). Thus, here, we will first describe briefly the method for evaluating (8.1) and then, at the end of this subsection we will describe the evaluation of (8.2).

The starting point is to let

$$q(\underline{n}) = I(\underline{n} \in \Omega) \prod_{k=1}^{K} q_k(n_k) \tag{8.3}$$

where $I(\cdot)$ is the indicator function and rewrite (8.1) as follows:

$$G = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \cdots \sum_{n_K=0}^{N_K} q(\underline{n}) \tag{8.4}$$

where $N_k = \max\{n_k : \underline{n} \in \Omega\}$. Thus calculating G involves a multi-dimensional summation.

First, we let $\underline{V}^i = (V_1^i, V_2^i, \ldots, V_K^i)$, $i = 1, 2, \ldots, n$, be a sequence of n i.i.d. random vectors, where each V^i takes values in $\Lambda = \{0, \ldots, N_1\} \times \{0, \ldots, N_2\} \times \ldots \times \{0, \ldots, N_K\}$. Next, we define $P_s(\underline{n}) = P(\underline{V}^i = \underline{n})$, for $\underline{n} \in \Lambda$, which is a sampling distribution that can be specified in a way that optimizes the efficiency of the Monte Carlo method, and set

$$Z^i = \frac{q(\underline{V}^i)}{P_s(\underline{V}^i)}. \tag{8.5}$$

Then the quantity

$$\overline{Z}_n = \frac{1}{n} \sum_{i=1}^{n} Z^i \tag{8.6}$$

provides an unbiased estimator for G (i.e., $E[\overline{Z}_n] = G$). Moreover, the Central Limit Theorem implies that, for large n,

$$P\left(|\overline{Z}_n - G| \leq \frac{c(\eta)\sigma_n(Z)}{\sqrt{n}}\right) = 1 - \frac{\eta}{2} \tag{8.7}$$

where $c(\eta)$ is the critical value of the standard normal distribution $N(x)$, that is, it satisfies

$$1 - \frac{\eta}{2} = \int_{-\infty}^{c(\eta)} N(x)dx \tag{8.8}$$

and $\sigma_n^2(Z)$ is the sample variance of Z^i, for $i = 1, \ldots, n$, i.e.,

$$\sigma_n^2(Z) = \frac{1}{n-1}\sum_{i=1}^{n}(Z^i - \overline{Z}_n)^2. \tag{8.9}$$

Notice that, for any fixed n (simulation size), $\overline{Z}_n$ is an estimate for G, whose accuracy can be assessed for the confidence interval $100(1-\eta)\%$ by

$$\left[\overline{Z}_n - \frac{c(\eta)\sigma_n(Z)}{\sqrt{n}}, \overline{Z}_n + \frac{c(\eta)\sigma_n(Z)}{\sqrt{n}}\right] \tag{8.10}$$

induced by (8.7). As the samples are being drawn, the sample variance can be calculated and the confidence intervals can be given explicitly. Furthermore, if greater accuracy is desired, more samples can be drawn, thereby decreasing the width of the confidence interval. This method is particularly well suited for optimization, as only rough estimates are needed for performance measures and gradients when the current solution is not close to optimal. Ross [7] actually shows that the gradients of the performance measures pertaining to voice can be obtained with little additional effort.

From (8.7) it is clear that the effectiveness of the Monte Carlo summation method depends on

1. the effort required to generate $\underline{V}^i$ from the distribution $P_s(\underline{n})$, $\underline{n} \in \Lambda$;
2. the effort required to evaluate the ratio $q(\cdot)/P_s(\cdot)$ during the sampling procedure;
3. the size of σ^2, the variance of Z^i.

To improve the efficiency of the method the following steps are usually taken. First, if the random variables $V_1^i, V_1^i, \ldots, V_K^i$ are independent (i.e., $F_s(\underline{n}) =$

$P_1(n_1)P_2(n_2)\ldots P_K(n_K)$), the $\underline{V}_i$ can be generated in a total of $O(K)$ time with the alias algorithm (e.g., see [16]); this computational effort is independent of the number of values the stochastic process (which in our example is link occupancy) can take on. This means that the method can handle networks with large link capacities. Second, selecting the appropriate sampling distribution $P_s(\underline{n})$, for $\underline{n} \in \Lambda$, can significantly reduce the variance σ^2. In particular, it is desirable to sample more frequently the points $\underline{n}$, at which $q(\underline{n})$ is important, which is typically done by considering functions $P_s(\cdot)$ that are similar to $q(\cdot)$. Ideally, one would like $q(\cdot)/P_s(\cdot)$ to be nearly constant; however, there exists a tradeoff between this similarity and the effort required to sample from $P_s(\cdot)$.

As we already mentioned, many performance measures of interest are given by nonlinear functions of normalization constants, and thus quantities of the form (8.2) must be evaluated. A natural estimate for Φ based on an n sample simulation is

$$\Phi_n = \frac{\sum_{i=1}^n Y^i}{\sum_{i=1}^n Z^i} \tag{8.11}$$

where

$$Y^i = f(\underline{V}^i)q(\underline{V}^i)/P_s(\underline{V}^i) \tag{8.12}$$

and

$$Z^i = q(\underline{V}^i)/P_s(\underline{V}^i). \tag{8.13}$$

Although Φ_n converges (almost surely) to Φ of (8.3), Φ_n has the undesirable property of being biased. Fortunately, this bias diminishes as n becomes large. It is also known [7] that the ratio estimator Φ_n can be made free of bias to order $1/n$ with a modification that requires an insignificant amount of additional CPU time. Moreover, the confidence interval for Φ can again be constructed (see [7]) as the sampling proceeds (i.e., on line) as follows. Let $\overline{Y}_n$ and $\sigma_n^2(Y)$ be the sample mean and variance associated with Y_i, $i = 1, \ldots, n$ [defined analogously to $\overline{Z}_n$ of (8.6) and $\sigma_n^2(Z)$ of (8.9)]. Furthermore let

$$\sigma_n^2(Y,Z) = \frac{1}{n-1}\sum_{i=1}^n (Y^i - \overline{Y}_n)(Z^i - \overline{Z}_n) \tag{8.14}$$

be the sample covariance associated with the two sets of random variables. Then the $(1-\eta)100\%$ confidence interval for Φ_n is

$$\left(\frac{\overline{Y}_n\overline{Z}_n - \frac{c^2(\eta)}{n}\sigma_n^2(Y,Z) - r_n}{\overline{Z}_n^2 - \frac{c^2(\eta)}{n}\sigma_n^2(Z)}, \frac{\overline{Y}_n\overline{Z}_n - \frac{c^2(\eta)}{n}\sigma_n^2(Y,Z) + r_n}{\overline{Z}_n^2 - \frac{c^2(\alpha)}{n}\sigma_n^2(Z)}\right) \tag{8.15}$$

where $c(\eta)$ is as defined above and r_n is given by

$$r_n = \sqrt{\left[\overline{Y}_n\overline{Z}_n - \frac{c^2(\eta)}{n}\sigma_n^2(Y,Z)\right]^2 - \left[\overline{Z}_n^2 - \frac{c^2(\eta)}{n}\sigma_n^2(Z)\right]\left[\overline{Y}_n^2 - \frac{c^2(\eta)}{n}\sigma_n^2(Y)\right]} \quad (8.16)$$

Note that the width of the confidence interval is $O(1/\sqrt{n})$.

8.2 Monte-Carlo Summation for the Probability of Voice Blocking

The method described in Section 8.1 for the estimator (8.5)-(8.6) of (8.1) and the ratio estimator (8.11)-(8.13) of (8.2) is directly applicable to the evaluation of the voice blocking probabilities of (3.12a) and (3.12b). We need only set $K = |\mathcal{P}|$, $q(\underline{n}) = P(\underline{N}^v)$ of (3.9), $k = p$ for $p \in \mathcal{P}$, $N_k = c_p/r_p$, and $q_k(n) = (\rho_p^v)^n/n!$ and estimate first the normalization constant G of (3.10). The key is of course the choice of the sampling function $P(\underline{n}) = P_s(\underline{n})$. In [7] it is suggested that we select

$$P_s(\underline{n}) = \frac{1}{G_s}\prod_{p\in\mathcal{P}}\frac{\gamma_p^{n_p^v}}{n_p^v!} \quad (8.17)$$

where

$$G_s = \prod_{p\in\mathcal{P}}\sum_{m=0}^{c_p/r_p}\frac{\gamma_p^m}{m!}. \quad (8.18)$$

The γ_p for $p \in \mathcal{P}$ are the important sampling parameters. These can be set as $\gamma_p = \rho_p^v$ or they can be selected according to a more tedious procedure (see [7]), which yields narrower confidence intervals. The estimator of (8.5)-(8.6) then takes the form

$$\hat{G} = \overline{Z}_n = \frac{1}{n}\sum_{i=1}^{n}\frac{q(\underline{V}^i)}{P_s(\underline{V}^i)} = \frac{G_s}{n}\sum_{i=1}^{n}\alpha^i I(\underline{V}^i \in \Omega^v) \quad (8.19)$$

where G_s is given by (8.18), Ω^v is defined by (3.11), and

$$\alpha^i = \prod_{p\in\mathcal{P}}\left(\frac{\rho_p^v}{\gamma_p}\right)^{V_p^i} \quad (8.20)$$

Once $\hat{G}$ is obtained (as a function of the vector of link capacities $\underline{c}$), the blocking probabilities B_l of (3.12a) and B_p of (3.12b) are obtained from those expressions where $\hat{G}$ is used instead of G.

Is is also possible to estimate B_p directly using the estimator (8.11)-(8.13) as

$$\hat{P}_p = \Phi_n = \frac{\sum_{i=1}^{n} Y^i}{\sum_{i=1}^{n} Z^i} = \frac{\sum_{i=1}^{n} \alpha^i I(\underline{V}^i \in \Omega_p^v)}{\sum_{i=1}^{n} \alpha^i I(\underline{V}^i \in \Omega^v)} \tag{8.21}$$

where Ω^v is given by (3.11) and Ω_p^v is obtained from (3.11) by replacing the $1 \times |\mathcal{L}|$ vector of link capacities $\underline{c}$ by the vector $\underline{c} - \underline{A}_p^T$ defined below (3.12b). The voice blocking probability P_l of (3.12a) can be estimated directly in the same manner. We may use (8.21) again with Ω_l^v instead of Ω_p^v in the numerator; Ω_l^v is obtained from Ω^v of (3.11) by replacing c_l by $c_l - 1$.

8.3 Monte-Carlo Summation for the Probability of Data Queueing

We rewrite $1 - Q_l$, where Q_l is the probability of data queued at link l ($l \in \mathcal{L}$) as follows,

$$1 - Q_l = \frac{\sum_{(\underline{N}^v, \underline{N}^s) \in \Lambda} f(\underline{N}^v, \underline{N}^s) \cdot q(\underline{N}^v, \underline{N}^s)}{\sum_{(\underline{N}^v, \underline{N}^s) \in \Lambda} q(\underline{N}^v, \underline{N}^s)} \tag{8.22}$$

where

$$q(\underline{N}^v, \underline{N}^s) = \prod_{p \in \mathcal{P}} \frac{(\rho_{1p}^v)^{n_p^v - n_p^s} (\rho_{2p}^v)^{n_p^s}}{(n_p^v - n_p^s)! n_p^s!} \cdot I[0 \leq \sum_{p \in \mathcal{P}_l} r_p n_p^v \leq c_l, l \in \mathcal{L}] \tag{8.23}$$

$$f(\underline{N}^v, \underline{N}^s) = \sum_{n_l^d = 0}^{c_l'} P(n_l^d | c_l') \cdot I[(\sum_{p \in \mathcal{P}_l} r_p n_p^v, \sum_{p \in \mathcal{P}_l} r_p n_p^s) \neq (c_l, 0)] \tag{8.24}$$

$$\Lambda = \{(\underline{N}^v, \underline{N}^s) | 0 \leq r_p n_p^v \leq c_p, 0 \leq n_p^s \leq n_p^v, p \in \mathcal{P}\}$$

and

$$\rho_{1p}^v = \frac{\beta}{\alpha + \beta} \cdot \rho_p^v \quad , \quad \rho_{2p}^v = \frac{\alpha}{\alpha + \beta} \cdot \rho_p^v \tag{8.25}$$

Then $1 - Q_l$ can be estimated by the ratio estimator of (8.11) as

$$\Phi_n = \frac{\sum_{i=1}^{n} Y^i}{\sum_{i=1}^{n} Z^i}$$

where

$$Y^i = \frac{f(\underline{V}^i, \underline{U}^i) \cdot q(\underline{V}^i, \underline{U}^i)}{P_s(\underline{V}^i, \underline{U}^i)} \tag{8.26}$$

$$Z^i = \frac{q(\underline{V}^i, \underline{U}^i)}{P_s(\underline{V}^i, \underline{U}^i)} \tag{8.27}$$

and $(\underline{V}^i, \underline{U}^i), i = 1, ..., n$ is a sequence of i.i.d. random vectors, where each $(\underline{V}^i, \underline{U}^i)$ takes values in Λ; $P_s(\underline{V}^i, \underline{U}^i)$ are importance sampling functions of the form

$$P_s(\underline{V}^i = \underline{N}^v, \underline{U}^i = \underline{N}^s) = \frac{1}{G_s} \cdot \prod_{p \in \mathcal{P}} \frac{\gamma_{1p}^{n_p^v - n_p^s} \gamma_{2p}^{n_p^s}}{(n_p^v - n_p^s)! n_p^s!}, \quad (\underline{N}^v, \underline{N}^s) \in \Lambda \tag{8.28}$$

where

$$G_s = \prod_{p \in \mathcal{P}} \sum_{n_p^v = 0}^{c_p/r_p} \sum_{n_p^s = 0}^{n_p^v} \frac{\gamma_{1p}^{n_p^v - n_p^s} \gamma_{2p}^{n_p^s}}{(n_p^v - n_p^s)! n_p^s!} = \prod_{p \in \mathcal{P}} \sum_{n_p^v = 0}^{c_p/r_p} \frac{\gamma_p^{n_p^v}}{n_p^v!} \tag{8.29}$$

and the importance sampling parameters γ_{1p} and γ_{2p} are given by

$$\gamma_{1p} = \frac{\beta}{\alpha + \beta} \cdot \gamma_p \quad , \quad \gamma_{2p} = \frac{\alpha}{\alpha + \beta} \cdot \gamma_p. \tag{8.30}$$

With the above definitions and the definitions of $\overline{Y}_n$, $\overline{Z}_n$, $\sigma_n^2(Y)$, $\sigma_n^2(Z)$, and $\sigma_n^2(Y, Z)$ of (8.6), (8.9), and (8.14), respectively, we obtain from (8.15) that the $1 - \eta$ confidence interval for $1 - \Phi_n$ the estimate of $1 - Q_l$ is

$$\left(1 - \frac{\overline{Y}_n \overline{Z}_n - \frac{c^2(\eta)}{n} \sigma_n^2(Y, Z) + r_n}{\overline{Z}_n^2 - \frac{c^2(\eta)}{n} \sigma_n^2(Z)}, 1 - \frac{\overline{Y}_n \overline{Z}_n - \frac{c^2(\eta)}{n} \sigma_n^2(Y, Z) - r_n}{\overline{Z}_n^2 - \frac{c^2(\eta)}{n} \sigma_n^2(Z)}\right) \tag{8.31}$$

where $c(\eta)$ and r_n are as defined in (8.8) and (8.16).

Notice that $(V_1^i, U_1^i), (V_2^i, U_2^i), ..., (V_{|\mathcal{P}|}^i, U_{|\mathcal{P}|}^i)$ are independent, i.e.,

$$P_s(\underline{V}^i = \underline{N}^v, \underline{U}^i = \underline{N}^s) = \prod_{p \in \mathcal{P}} P_s(V_p^i = n_p^v, U_p^i = n_p^s) \tag{8.32}$$

and

$$P_s(V_p^i = n_p^v, U_p^i = n_p^s) = P_s(V_p^i = n_p^v) \cdot P_s(U_p^i = n_p^s | V_p^i = n_p^v) \quad , \tag{8.33}$$

where

$$P_s(V_p^i = n_p^v) = \frac{\frac{\gamma_p^{n_p^v}}{n_p^v!}}{\sum_{n_p^v = 0}^{c_p/r_p} \frac{\gamma_p^{n_p^v}}{n_p^v!}}, \quad 0 \le r_p n_p^v \le c_p \tag{8.34}$$

and

$$P_s(U_p^i = n_p^s | V_p^i = n_p^v) = \frac{\frac{\beta^{n_p^v - n_p^s} \alpha^{n_p^s}}{(\alpha+\beta)^{n_p^v}} \cdot n_p^v!}{(n_p^v - n_p^s)! n_p^s!}, \quad 0 \le n_p^s \le n_p^v \tag{8.35}$$

Therefore, $|\mathcal{P}|$ of (V_p^i, U_p^i) can be generated independently by first generating V_p^i from $P_s(V_p^i)$ and then generating U_p^i from $P_s(U_p^i|V_p^i)$. The alias algorithm is used for both generating processes of V_p^i and U_p^i.

In our numerical results we simulate for a 95% confidence interval with $n = 300,000$ based on the importance sampling scenario $\gamma_p = \rho_p^v, p \in \mathcal{P}$.

8.4 Monte-Carlo Summation for the Average Queueing Data Delay

This is the same as for the probability of data queueing except for the different function $f(\underline{N}^v, \underline{N}^s)$. The new function is given by (7.5)-(7.6) in Section 7.

9. ADMISSION CONTROL VIA REDUCED-LOAD APPROXIMATIONS

Admission control strategies can play an important role in integrated voice/data networks because, by controlling in a coordinated manner the admission of new calls in all network circuits, the overall probability of voice blocking can be reduced and so may the probability of queueing data and the data packet delay. In this paper admission control strategies based on (a) thresholds on the individual path traffic and (b) linear combinations of voice traffic over selected sets of paths are considered. The justification for this is two-fold: First, such admission control strategies have been shown in [11]-[12] to work well for voice-only multi-hop radio networks of small size. Second, this type of admission control strategies enables the extension of the approximations of this report from problems without control to problems with control at the expense of only minimum additional computational complexity. Therefore, since our approximations can handle mid-size and even large-size multi-hop radio networks, near-optimal control strategies can be derived for networks of any size based on the approximate performance measures evaluated below.

After describing the relevant models for admission control in Section 9.1, we present in Sections 9.2 and 9.3 two distinct methods for incorporating admission control strategies into the approximation methods outlined in Sections 4 (for the probability of voice blocking) and in 5 and 6 (for the probability of data queueing). The development in these sections appears for the first time in this report. Only the modification of the knapsack approximation is described in detail. The Pascal approximation can be also modified in a similar way but this is omitted from the report.

9.1 Models for Admission Control

The problem of admission control is formulated in the context of our exposition in the previous sections, as follows. Together with the **resource (bandwidth) constraints**

$$\sum_{p\in\mathcal{P}_l} r_p n_p^v \le c_l, \qquad l \in \mathcal{L} \tag{9.1}$$

[where $r_p = 1$ for all $p \in \mathcal{P}$ and c_l, $\mathcal{P}_l$, $l \in \mathcal{L}$, must be replaced by T_n, $\mathcal{P}_n$ for $n \in \mathcal{N}$ for the radio network of Section 2.1] which have played an important role in the previous analysis, we also consider the additional **control constraints**

$$\sum_{p\in\mathcal{P}_s} r_p^{(s)} n_p^v \le Y_s, \qquad s \in S. \tag{9.2}$$

These control constraints result in the blocking of voice calls, even when network resources are available to support them. We refer to a system without such control constraints (i.e., one in which only the resource constraints of Eq. (9.1) are applicable) as an "uncontrolled" system. In Eq. (9.2) S is the set of all such constraints, Y_s is the threshold for the s-th constraint, and $\mathcal{P}_s$ is the set of paths involved in the constraint. These control constraints couple the voice traffic over selected sets of paths. Moreover, **direct threshold constraints** of the form

$$n_p^v \leq X_p, \qquad p \in \mathcal{P} \tag{9.3}$$

can be considered as special cases of the more general constraint of (9.2). Also notice that the quantities $r_p^{(s)}$ for $p \in \mathcal{P}_s$ involved in the s-th constraint need not be equal to (the data rate) r_p nor do they need to be equal to each other. Clearly, with this model any linear-combination type of constraint on the number of calls of classes $p \in \mathcal{P}$ can be considered. The objective is to determine the set of constraints (which collectively constitute a control policy) that results in optimal performance.

9.2 The Knapsack Approximation via Conditioning on Additional Constraints

We now show how to incorporate one additional control constraint into the knapsack approximation, which is obtained under the standard resource constraint of (9.1). The procedure can be extended to apply to two or more control constraints at the expense of an increasing computational effort due to the dimensionality of the problem.

For this section we assume that $S = \{s\}$ (a single-element set), that Y_s is the control threshold [it can actually be $Y_s/r^{(s)}$ for $r_p^{(s)} = r^{(s)}$ ($p \in \mathcal{P}_s$), when all $r_p^{(s)}$ are equal to each other], and that c_l is the capacity of the l-th link ($l \in \mathcal{L}$). We rewrite the two constraints (the resource constraint for link l, and the control constraint with threshold Y_s) as

$$0 \leq \sum_{p \in \mathcal{P}_l} r_p n_p^v = k_1 \leq c_l \tag{9.4}$$

$$0 \leq \sum_{p \in \mathcal{P}_s} r_p^{(s)} n_p^v = k_2 \leq Y_s \tag{9.5}$$

in terms of the quantities k_1 and k_2 representing the sums in (9.4) and (9.5), respectively.

9.2.1 Probability of Voice Blocking

Application of the knapsack approximation to the above formulation yields

$$K_{lp} = 1 - \sum_{k_1=0}^{c_l - r_p} \sum_{k_2=0}^{Y_s - r_p^{(s)}} P(k_1, k_2) \tag{9.6}$$

(for $p \in \mathcal{P}_l \cap \mathcal{P}_s$) as the voice blocking probability where

$$P(k_1, k_2) = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k_1, k_2)} P(\underline{N}^v, \underline{N}^s) \tag{9.7}$$

$$\Omega(k_1, k_2) = \left\{ (\underline{N}^v, \underline{N}^s) | 0 \le n_p^s \le n_p^v, p \in \mathcal{P}_l \cup \mathcal{P}_s; 0 \le \sum_{p \in \mathcal{P}_l} r_p n_p^v = k_1 \le c_l; 0 \le \sum_{p \in \mathcal{P}_s} r_p^{(s)} n_p^v = k_2 \le Y_s \right\} \tag{9.8}$$

and $\sum_{k_1=0}^{c_l} \sum_{k_2=0}^{Y_s} P(k_1, k_2) = 1$. After the normalization $P'(k_1, k_2) = P(k_1, k_2)/P(0,0)$, (9.6) becomes

$$K_{lp} = \begin{cases} 1 - \frac{\sum_{k_1=0}^{c_l - r_p} \sum_{k_2=0}^{Y_s - r_p^{(s)}} P'(k_1, k_2)}{\sum_{k_1=0}^{c_l} \sum_{k_2=0}^{Y_s} P'(k_1, k_2)}; & \text{if } p \in \mathcal{P}_l \cap \mathcal{P}_s \\ 1 - \frac{\sum_{k_1=0}^{c_l - r_p} \sum_{k_2=0}^{Y_s} P'(k_1, k_2)}{\sum_{k_1=0}^{c_l} \sum_{k_2=0}^{Y_s} P'(k_1, k_2)}; & \text{if } p \in \mathcal{P}_l \cap \mathcal{P}_s^c \end{cases} \tag{9.9}$$

Proposition 9.1

$P'(k_1, k_2)$ satisfies the recursion

$$P'(k_1, k_2) = \begin{cases} \frac{1}{k_1} \cdot \frac{\beta}{\alpha+\beta} \left[\sum_{p \in \mathcal{P}_l \cap \mathcal{P}_s^c} r_p \rho_p^v P'(k_1 - r_p, k_2) + \sum_{p \in \mathcal{P}_l \cap \mathcal{P}_s} r_p \rho_p^v P'(k_1 - r_p, k_2 - r_p^{(s)}) \right]; & \\ & \text{if } r_p \le k_1 \le c_l, r_p^{(s)} \le k_2 \le Y_s \\ \frac{1}{k_1} \cdot \frac{\beta}{\alpha+\beta} \sum_{p \in \mathcal{P}_l \cap \mathcal{P}_s^c} r_p \rho_p^v P'(k_1 - r_p, k_2); & \\ & \text{if } r_p \le k_1 \le c_l, k_2 = 0 \\ \frac{1}{k_2} \cdot \frac{\beta}{\alpha+\beta} \sum_{p \in \mathcal{P}_s \cap \mathcal{P}_l^c} r_p^{(s)} \rho_p^v P'(k_1, k_2 - r_p^{(s)}) & \\ & \text{if } k_1 = 0, r_p^{(s)} \le k_2 \le Y_s \\ 1; & \text{if } k_1 = 0,\ k_2 = 0 \\ 0; & \text{if } k_1 \text{ is not an integer multiple of } r_p \\ & \text{or } k_2 \text{ is not an integer multiple of } r_p^{(s)} \end{cases} \tag{9.10}$$

where $r_p \leq k_1 \leq c_l$ and $r_p^{(s)} \leq k_2 \leq T_s$.

Proof: It follows easily as a special case of of Appendix B; we only need to set $r_p^{(1)} = r_p$, $r_p^{(2)} = r_p^{(s)}$, $\mathcal{P}_1 = \mathcal{P}_l$, $\mathcal{P}_2 = \mathcal{P}_s$, $Z_1 = c_l$, $Z_2 = Y_s$, and use only the constraints on k_i; we set $m_i = 0$ for $i = 1, 2$.

9.2.2 Probability of Data Queueing

The application of the knapsack approximation to the probability of queueing data when conditions (9.1) and (9.2) are present requires an extension of the method of Section 5.2 to incorporate additional dimensions. In particular, instead of the k_i, m_i for 1,2 defined and used in Section 5.2 to represent the necessary integer entities for nodes $n_1 = 1$ and $n_2 = 2$ (connected by link l) and which reflect only the bandwidth (or transceiver) conditions, we must introduce additional such entities to represent the threshold condition of (9.2). Actually, we need a total of six integers $(k_1, m_1; k_2, m_2; k_3, m_3)$ to represent all necessary entities: (k_1, m_1) for the transceiver constraint on the set $\mathcal{P}_1 \cap \mathcal{P}_s$, (k_2, m_2) for the transceiver constraint on the set $\mathcal{P}_2 \cap \mathcal{P}_s$, and (k_3, m_3) for the threshold constraint on the set $\mathcal{P}_s$; compare with (9.4)-(9.5) and (5.15). This requires the definition of a six-dimensional entity $P(k_1, m_1; k_2, m_2; k_3, m_3)$ similar to that of (5.16) and the corresponding recursion. The details are complicated but straightforward and are *omitted*. The rest of the knapsack approximation for this case proceeds as in Section 5.2.

9.2.3 Extension to Multiple Control Constraints

The formulation, the knapsack approximations, and the recursions for the probabilities of voice blocking and data queueing can be extended to include additional control constraints of the form (9.2) for several distinct $s \in \mathcal{S}$. For a small cardinality number $|\mathcal{S}|$ the computational effort, although increasing exponentially (as $2^{|\mathcal{S}|}$) with the total number of control constraints, remains reasonable; however, for large $|\mathcal{S}|$ it becomes prohibitive. This is the reason for our consideration in Section 9.3 of a different approach that requires a substantially reduced computational effort.

9.3 The Knapsack Approximation via the Introduction of Fictitious Links

The second method for applying the knapsack approximation to integrated voice/data networks with admission control is very straightforward and does not suffer from the computational problems of the first. Actually, the required computational effort grows only linearly with the number of control constraints. The key idea here is to introduce fictitious links l_s with link capacity Y_s that represent each of the control constraints ($s \in \mathcal{S}$)

of (9.2). On these fictitious links the paths carry information at a rate $r_p^{(s)}$ (which may not be equal to r_p). In this manner all voice calls transmitted over the paths (circuits) which satisfy a particular control constraint (i.e., $p \in \mathcal{P}_s$) use this fictitious link, that is $l_s \in p$ for all $p \in \mathcal{P}_s$. This means that besides the natural links of the network (part of the given architecture) new links are added which do not represent any physical connection but rather they signify satisfaction of control constraints. After the set of links $\mathcal{L}$ has been expanded to include the new links, that is $\mathcal{L}' = \mathcal{L} \cup \mathcal{S}$, we apply the usual procedure for the knapsack approximation to the new expanded set of links $\mathcal{L}'$. For example for the voice blocking probability we use the recursions

$$L_{l'q} = K_{l'q}\left(c_{l'}; \rho_p^v \prod_{\ell' \in p, \ell' \neq l'} (1 - L_{\ell' p}),\ p \in \mathcal{P}_{l'},\ l' \in \mathcal{L}' = \mathcal{L} \cup \mathcal{S}\right)$$

for $q \in \mathcal{P}_{l'}$, where $\mathcal{P}_{l'} = \{p \in \mathcal{P} | l' \in p, l' \in \mathcal{L}'\}$ and finally $B_q = 1 - \prod_{l' \in q}(1 - L_{l'q})$. In computing $K_{l'q}$ [for example via (4.5)-(4.6)] we must be careful to use (c_l, r_p) for the real links (when $l' \in \mathcal{L}$) and $(Y_s, r_p^{(s)})$ for the fictitious links (when $l' \in \mathcal{S}$). The above approach is also applicable to the approximation of the probability of data queueing and the average data delay (Section 5) after the expansion of $\mathcal{L}$ to $\mathcal{L}'$ has taken place.

10. REVENUE SENSITIVITY VIA REDUCED-LOAD APPROXIMATIONS

In several practical problems of performance evaluation and optimization in networks such as optimal data routing and the allocation of additional resources in response to increases in traffic demand, the rates of change or derivatives (termed sensitivities) of certain performance measures (termed revenue) with respect to network resources (e.g., link capacities) and traffic loads (e.g., average offered voice or data traffic) play an important role. If accurate approximations to these sensitivities that require modest computational effort for their evaluation can be derived, then they can be used in a variety of optimization problems to derive near-optimal control or allocation strategies. This is exactly what is accomplished in this section.

Specifically, we first present certain popular measures of revenue and then derive their (approximate) sensitivities with respect to link capacities, voice loads, and data loads. This is first accomplished in Section 10.1 for the general multi-rate network model (of Sections 2.1.1 and 2.2.1); in particular, Section 10.1.1 deals with suitable sensitivities for voice revenue and Section 10.1.2 deals with suitable sensitivities for data revenue. Then in Section 10.2 we outline how to modify and apply these results to the multi-hop radio network model of Sections 2.1.2 and 2.2.2.

10.1 Revenue Sensitivities for General Multi-Rate Networks

Suitable choices for the long-run average voice and data revenues for the multi-rate network of Sections 2.1.1 and 2.2.1 are

$$W^v(\underline{\rho}^v;\underline{c}) = \sum_{p\in\mathcal{P}} \gamma_p^v \rho_p^v (1 - B_p) \tag{10.1}$$

$$W^d(\underline{\rho}^d,\underline{\rho}^v;\underline{c}) = \sum_{l\in\mathcal{L}} \gamma_l^d \rho_l^d (1 - Q_l), \tag{10.2}$$

where γ_p^v ($p \in \mathcal{P}$) and γ_l^d ($l \in \mathcal{L}$) is the revenue rate when a path-p call is accepted and link-ℓ data traffic is served, respectively. Thus (10.1) and (10.2) provide natural definitions of the (long-term) average revenue generated when voice calls are accepted (not blocked) and data packets are served (not queued), respectively. In the above notation $\underline{\rho}^v$ and $\underline{\rho}^d$ denote the vectors of voice and data loads ($\rho_p^v, p \in \mathcal{P}$ and $\rho_l^d, l \in \mathcal{L}$) and $\underline{c}$ the vector of link capacities ($c_l, l \in \mathcal{L}$). In this section it is assumed that the probabilities of voice blocking B_p and data queueing Q_l are evaluated via the knapsack approximation.

10.1.1 Sensitivity of Voice Revenue

The sensitivity of the voice revenue with respect to link capacities and voice loads is evaluated in [6] (see also [7]) via the knapsack and Pascal approximation methods. Here we repeat the results for the sake of completeness and to introduce the necessary notation.

For the voice revenue the following sensitivity measures are defined with respect to link capacity and voice loads:

$$c_{lp}^{v} \simeq W^{v}\left(\underline{\rho}^{v};\underline{c}-r_{p}\underline{e}_{p}\right)-W^{v}(\underline{\rho}^{v};\underline{c}) \tag{10.3}$$

$$\frac{dW^{v}}{d\rho_{p}^{v}} \simeq (1-B_{p})\cdot(\gamma_{p}^{v}-\sum_{l\in p}c_{lp}^{v}) \tag{10.4}$$

where $\underline{e}_p$ is an $|\mathcal{L}|\times 1$ vector with unit value at entries $l \in p$ and zero value at all others.

The quantities c_{lp}^{v} in (10.3) have been called shadow prices in [6] and represent the expected revenue lost (number of blocked voice calls) when we remove r_p circuits from link l (or accept a call of path p at link l) for one unit of time. If we make the approximation that class-q calls arrive at link l according to a Poisson process with offered load

$$\bar{\rho}_{q}^{v}=\rho_{q}^{v}\prod_{\ell\in q,\ell\neq l}(1-L_{\ell q})$$

(termed thinned load, with $L_{\ell q}$ as defined in Section 4.2), then the expected loss in revenue from class-q connections being blocked at link l due to the removal of r_p circuits for one unit of time is $\bar{\rho}_q^v \cdot h_{lqp}\cdot\gamma_q$, where

$$h_{lqp}=K_{lq}\left(c_{l}-r_{p},\rho_{q'}^{v}\prod_{\ell\in q',\ell\neq l}(1-L_{\ell q}),q'\in\mathcal{P}_{l}\right)-K_{lq}\left(c_{l},\rho_{q'}^{v}\prod_{\ell\in q',\ell\neq l}(1-L_{\ell q'}),q'\in\mathcal{P}_{l}\right) \tag{10.5}$$

where K_{lq} is given by (4.5)-(4.6). However, each class-q connection blocked on link l (if it had been accepted), would have used r_q circuits on each link $\ell\in q,\ell\neq l$. Thus the expected gain in revenue from additional connections being accepted on links $\ell\in q,\ell\neq l$ due to a class-q connection being blocked at link l for this one unit of time is $\bar{\rho}_q^v\cdot h_{lqp}\cdot\sum_{j\in q,j\neq l}c_{jq}^v$. Subtracting the gain from the loss and summing over $q\in\mathcal{P}_l$ yields the following system of linear equations:

$$c_{lp}^{v}\simeq\sum_{q\in\mathcal{P}_{l}}\left[\bar{\rho}_{q}^{v}K_{lq}\left(c_{l}-r_{p},\bar{\rho}_{q'}^{v},q'\in\mathcal{P}_{l}\right)-\bar{\rho}_{q}^{v}K_{lq}\left(c_{l},\bar{\rho}_{q'}^{v},q'\in\mathcal{P}_{l}\right)\right]\cdot\left(\gamma_{q}-\sum_{\ell\in q,\ell\neq l}c_{\ell q}^{v}\right),$$

$$\simeq \sum_{q \in \mathcal{P}_l} \left[\rho_q^v (1 - B_q(\underline{c})) - \rho_q^v \left(1 - B_q\left(\underline{c} - r_p \underline{e}_p\right)\right)\right] \cdot \left(\gamma_q - \sum_{\ell \in q, \ell \neq l} c_{\ell q}^v\right). \tag{10.6}$$

In deriving (10.6) we first use the results of Section 4.2 to show that the relationship between the load ρ_q^v and the thinned load $\bar{\rho}_q^v$ on link l is

$$\bar{\rho}_q^v(1 - L_{lq}) = \rho_q^v(1 - B_q)$$

and from this it follows that

$$\bar{\rho}_q^v K_{lq}\left(c_l, \bar{\rho}_{q'}^v, q' \in \mathcal{P}_l\right) = \bar{\rho}_q^v L_{lq} = \bar{\rho}_q^v - \rho_q^v\left(1 - B_q(\underline{c})\right).$$

Finally, the right hand side term in (10.4) can be interpreted as follows. An additional call offered to route p will be accepted with probability $(1 - B_p)$; if accepted it will earn γ_p^v revenue, but at a cost c_{lp} for each link $l \in p$.

10.1.2. Sensitivity of Data Revenue

In contrast to the previous section that reviewed the results of [6] for the sensitivities of the voice revenue, the derivation of sensitivities for the data revenue of integrated voice/data multi-rate networks appears for the first time in this section. The following sensitivity measures with respect to link capacities and data loads, respectively, are suggested for the data revenue of (10.2)

$$c_l^d \simeq W^d(\underline{\rho}^v, \underline{\rho}^d; \underline{c}) - W^d(\underline{\rho}^v, \underline{\rho}^d; \underline{c} - \underline{e}_l) \tag{10.7}$$

$$\frac{\partial W^d}{\partial \rho_l^d} \simeq (1 - Q_l) \cdot (\gamma_l^d - c_l^d) \tag{10.8}$$

where $\underline{e}_l$ is an $|\mathcal{L}| \times 1$ vector with unit value at the entry l (the link in question) and zero value at all other entries.

Since link data behavior is assumed to be mutually independent (refer to Section 2.2.1), the term in (10.7) can be obtained in the form

$$c_l^d \simeq \left[\rho_l^d(1 - Q_l(\underline{c})) - \rho_l^d(1 - Q_l(\underline{c} - \underline{e}_l))\right] \cdot \gamma_l^d \tag{10.9}$$

following similar reasoning to that which resulted in (10.6) above for the voice revenue shadow prices. However, because of the inter-link independence assumption the approximation in (10.9) can not be fully trusted.

The term in (10.8) again can be obtained intuitively by noting that additional data offered to link l will not be queued with probability $(1 - Q_l)$; if accepted, it will earn γ_ℓ revenue but at a cost c_l^d.

Instead of obtaining $\partial W^d/\partial\rho_\ell^d$ from (10.8) for which (10.9) is necessary, an alternative way of computing $\partial W^d/\partial\rho_\ell^d$ is used in this report, according to which

$$\frac{\partial W^d}{\partial \rho_\ell^d} \simeq \gamma_l^d(1 - Q_l) + \gamma_l^d\rho_l^d\frac{\Delta(1 - Q_l)}{\Delta\rho_l^d} \tag{10.10}$$

where $\Delta(1 - Q_l)/\Delta\rho_l^d$ denotes the ratio of finite differences for $\Delta\rho_l^d = .001$.

The evaluation of the data revenue sensitivity with respect to voice load is considerably more complicated. The result is given by the following Proposition:

Proposition 10.1

The data revenue sensitivity with respect to voice loads takes the form

$$\frac{\partial W^d}{\partial \rho_p^v} = (1 - B_p)\left\{\frac{\beta}{\alpha+\beta}\cdot W^d(\underline{c} - r_p\underline{e}_p) - W^d(\underline{c})\right.$$

$$\left. + \frac{\alpha}{\alpha+\beta}\cdot\frac{1}{G(\underline{c} - r_p\underline{e}_p)}\sum_{(\underline{N}^v,\underline{N}^s),\in\Omega(\underline{c}-r_p\underline{e}_p)}\prod_{p\in\mathcal{P}}\rho_p^{n_p^v}\frac{\frac{\alpha^{n_p^s}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v}}}{n_p^s!(n_p^v - n_p^s)!}\cdot w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c})\right\}$$

$$= (1 - B_p)\left[\frac{\beta}{\alpha+\beta}\cdot W^d(\underline{c} - r_p\underline{e}_p) - W^d(\underline{c})\right.$$

$$\left. + \frac{\alpha}{\alpha+\beta}\cdot\sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c}-r_p\underline{e}_p)} P_{(\underline{c}-r_p\underline{e}_p)}(\underline{N}^v,\underline{N}^s)\cdot w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c})\right]. \tag{10.11}$$

where G is given by (3.3), $\bar{P}(c_l', \rho_l^d)$ is given by (5.6), the residual capacity for data is $c_l' = c_l - \sum_{p\in\mathcal{P}_l} r_p(n_p^v - n_p^s)$, and

$$w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c}) = \sum_{l\in\mathcal{L}}\gamma_l^d\rho_l^d\cdot\bar{P}(c_l',\rho_l^d)\cdot I(c_l' > 0). \tag{10.12}$$

The terms in (10.11) and (10.12) can be approximated by applying the knapsack or Pascal methods. In particular, the knapsack approximation is applied to the Q_l terms in

$$W^d(\underline{c} - r_p\underline{e}_p) = \sum_{l\in\mathcal{L}}\gamma_l^d\rho_l^d(1 - Q_l(\underline{c} - r_p\underline{e}_p)), \tag{10.13}$$

$$W^d(\underline{c}) = \sum_{l\in\mathcal{L}}\gamma_l^d\rho_l^d(1 - Q_l(\underline{c})), \tag{10.14}$$

and to the term

$$\sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c}-r_p\underline{e}_p)} P_{(\underline{c}-r_p\underline{e}_p)}(\underline{N}^v,\underline{N}^s)w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c})$$

$$=\sum_{l\in\mathcal{L}}\gamma_l^d\rho_l^d \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c}-r_p\underline{e}_p)} P_{(\underline{c}-r_p\underline{e}_p)}(\underline{N}^v,\underline{N}^s)\cdot\sum_{n_l^d=0}^{c_l'-1}P(n_\ell^d|c_\ell')\cdot I(c_\ell'>0). \quad (10.15)$$

Proof: Provided in Appendix H

10.2. Revenue Sensitivities for the Multi-Hop Radio Network of Sections 2.1.2 and 2.2.2

Recall that for the multi-hop radio network of Section 2.1.2 and 2.2.2, the link capacity vector $\underline{c}$ should be replaced by the node transceiver vector $\underline{T}$ and the rate of voice calls in all circuits is $r_p = 1,\ p\in\mathcal{P}$.

To obtain the sensitivities of the voice revenue we must replace $\underline{c}$ by $\underline{T}$, c_l by T_n, $l\in\mathcal{L}$ (links) by $n\in\mathcal{N}$ (nodes), $\mathcal{P}_l$ (set of all circuits or paths using link l) by $\mathcal{P}_n$ (set of all paths passing by node n), and $\underline{c}-r_p\underline{e}_p$ by $\underline{T}-\underline{e}_p$ (where $\underline{e}_p$ is an $|\mathcal{N}|\times 1$ vector with unit value at entries $n\in p$ and zero value at all others) in (10.3), (10.4), (10.6) and all other expressions of Section 10.1.1.

To obtain the sensitivities of the data revenue we must carry out the same substitutions in (10.10) and in (10.11)-(10.14) as we did for the voice revenue sensitivities in the previous paragraph. In (10.11)-(10.14) the following additional substitution is necessary: the residual (data) capacity

$$c_l' = c_l - \sum_{p\in\mathcal{P}_l} r_p(n_p^v - n_p^s)$$

of link $l\in\mathcal{L}$ connecting nodes $n_1,n_2\in\mathcal{N}$ [i.e., $l=(n_1,n_2)$] must be replaced by

$$c_l' = \min\{T_{n_1} - \sum_{p\in\mathcal{P}_{n_1}}(n_p^v-n_p^s)\ \ ,\ \ T_{n_2}-\sum_{p\in\mathcal{P}_{n_2}}(n_p^v-n_p^s)\}.$$

11. NUMERICAL RESULTS AND PERFORMANCE COMPARISONS

The presentation of the numerical results in this section is organized as follows. In Section 11.1 the network paradigms used for general multi-rate wired networks and for a radio multi-hop network are described in detail in terms of architectures, network parameters, and traffic parameters. In Section 11.2 comparisons of the approximations to and the exact values of the performance measures of interest are carried out for the radio network paradigm with no admission control. This is repeated in Section 11.3 for the general multi-rate multi-hop wired network paradigm. Admission control schemes for voice traffic (based on thresholds) in the radio network paradigm are described in Section 11.4. Section 11.5 presents the approximations to the voice and data revenue sensitivities for the radio and wired network paradigms of Section 11.1. Finally, Section 11.6 discusses the required computational effort for the various approximations.

11.1 Network and Traffic Models for the Paradigms Used

The two networks of interest are shown in Figures 2a-2b and 3a-3b. The network of Figure 2a is the paradigm shown in [11]-[12]. It is a ten-node multi-hop radio network, in which the main resource of interest is the number of transceivers at each of the network nodes. In most of our examples involving this network, $\underline{T}$ (which denotes the vector of transceivers at the ten network nodes) takes the value $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$; we also study $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$ to demonstrate the quality of the approximations when applied to a smaller system. In Figure 2b, the five circuits used by voice calls in the network of Figure 2a are shown. The voice calls follow the model of Sections 2.2.1 and 2.2.2 and have in all examples (unless specified otherwise) activity factor $\frac{\beta}{\alpha+\beta} = .4$ (half-duplex); however; limited results are also presented for voice activity factors of 0.8 (full-duplex) and 1.0 (corresponding to a voice model with no silence periods). The network data traffic is transmitted over the same nine links, that are used by the aforementioned five voice circuits. As discussed in Section 2.2.2, an M/D/c model is used for data traffic over the above nine links, with capacities defined by Eq. 2.0. In this model, the data can only use transceivers that are not occupied by voice calls. In the model of Section 7.3, some transceivers at every node are a priori dedicated to data traffic and the data link capacity is defined by (7.18); we will come back to this model when we show results for the average data delay. The network of Figures 2a-2b is characterized by a single data rate, which assumes a common value for both voice and data traffic.

Figures 3a and 3b, show the multi-rate star network of [6], which is used in this section as a paradigm of a general-purpose integrated voice/data multi-rate wired network. The main network resource is the vector of link capacities $\underline{c}$. Initially we reproduce the example of [6] by setting $\underline{c} = (90, 100, 110, 120)$ for the four links of the network; later we demonstrate the accuracy of the approximations for smaller values of the link capacities as well. There are twelve voice circuits in the network of Figure 3a; and Figure 3b lists the links used and the transmission bandwidth required for each voice circuit. Data traffic is assumed to always require unit transmission bandwidth. The voice model is described in Section 2.2.1 and the activity factor for the voice calls is $\frac{\beta}{\alpha+\beta} = .4$. An M/D/c model is again used for data traffic with link capacity given by (3.7b). In this model, data traffic is allowed to use only the capacity left unused in the four network links after the voice call requests have been accommodated. In the model of Section 7.3, some portion of the capacity of each link is a priori dedicated to data traffic and the data link capacity is given by (7.17); we will come back to this model when we show results for the average data delay.

11.2 Comparisons of Approximations for the Radio Network of Figure 2 without Admission Control

This group of numerical results pertains to the radio network model of Figure 2 in the absence of admission control, i.e., a voice call is admitted if and only if a transceiver is available at every node along the predetermined path. Networks with eight transceivers per node are evaluated in Section 11.2.1 and networks with four transceivers per node are evaluated in Section 11.2.2. In these two subsections, the entire number of transceivers at each node is available for use by the voice traffic; data traffic is served only if some transceivers remain unoccupied. However, in Section 11.2.3 we also evaluate networks for which one pair of transceivers is dedicated to data traffic for each of the nine data links in the network of Figure 2.

11.2.1 Results for a Radio Network with Eight Transceivers per Node

We first consider the case of $\underline{T} = (8, 8, 8, 8, 8, 8, 8, 8, 8, 8)$, i.e., each node has eight transceivers. Figures 4 and 5 show the exact value, knapsack approximation, and Monte Carlo summation (the midpoint of the confidence interval is shown) for the average probabilities of voice blocking and data queueing, versus the offered voice and data loads, respectively. The "exact value" solutions for the probability of voice blocking are based on the product-form solution. Those for data packet queueing are based on the M/D/c model and Kleinrock's independence assumption, thus in the latter case the results are not truly exact but rather they are based on closed-form approximations. In both figures, the probabilities of voice blocking and data queueing represent the averages of such probabilities over all voice circuits and data links in the radio network, respectively, that is,

$$\bar{B} = \frac{\sum_{p \in \mathcal{P}} \rho_p^v B_p}{\sum_{p \in \mathcal{P}} \rho_p^v}$$

and

$$\bar{Q} = \frac{\sum_{l \in \mathcal{L}} \rho_l^d Q_l}{\sum_{l \in \mathcal{L}} \rho_l^d}.$$

In Figure 4, the value of the data load is irrelevant (since voice has priority over data) and the offered voice load $\rho_p^v r_p$ ranges from 0 to 15. In this case, we have a single-rate network with $r_p = 1$ for all paths, and ρ_p^v takes the same value for all five paths (voice circuits). Notice that the knapsack approximation, the Monte Carlo summation method, and the exact expression for the probability of voice blocking yield results that are very close to each other for all values of the offered voice load. Recall that the knapsack and Pascal approximations yield identical results for single-rate networks.

In deriving the approximation via the Monte Carlo summation method, we used 300,000 sample points (calls to the random number generator computer routines), which resulted in a confidence interval of 95%. Importance sampling scenarios according to which $\gamma_p = \rho_p^v$ ($p \in \mathcal{P}$) for the probability of voice blocking (refer to Section 8.2) and $\gamma_{1p} = \rho_p^v \beta/(\alpha + \beta)$, $\gamma_{2p} = \rho_p^v \alpha/(\alpha + \beta)$ ($p \in \mathcal{P}$) for the probability of data queueing (refer to Section 8.3) were used. Recall from the discussion in Section 8 [following equations (8.7)-(8.10) and (8.14)-(8.15)] that the confidence interval parameter η and the number of samples (calls of the random number generator) n are related in a complicated manner [e.g., see (8.10)], which involves the sample variance $\sigma_n(Z)$; therefore, the appropriate value of n that guarantees the desirable confidence interval is found through trial and error. The number 300,000 given above represents the result of several such attempts to find a number that is sufficiently large to work for most situations of interest (for all different values of the sample variances for the numerical examples considered). The variation of the 95% confidence interval did not exceed 2% of its mean value during this search.

Similar results are shown in Figure 5, in which the offered voice load is $\rho_p^v r_p = 5.5$ for all voice circuits, and the offered data load ρ_l^d ranges from 0 to 4. The data loads of all nine links are assumed equal. Again, the knapsack approximation is very close to the exact value of the probability of data queueing.

Tables 1-4 amplify the results of Figures 4 and 5 by providing detailed comparisons of exact values, knapsack approximation, and Monte Carlo summation for the probabilities of voice blocking and data queueing for a broad range of offered voice and data loads. The column denoted "percent error" refers to the percent relative error generated via a comparison of the results obtained using the knapsack approximation to the exact values [= 100 x (knapsack-exact)/(exact)]; the sign of the relative error has been maintained in these calculations. The results are organized in a manner that first presents the results for the average quantities (averaged over the traffic loads of all paths or links) and then the individual results for each path or link. This organization plan is followed for basically all the numerical results presented in tabular form in this report. Table 1 shows the average probability of voice blocking $\bar{B}$ (over all paths) for different average voice loads $\bar{\rho}^v$ defined as

$$\bar{\rho}^v = \frac{1}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} r_p \rho_p^v.$$

Then Table 2 shows the individual probabilities of voice blocking B_p for each of the paths p ($p \in \mathcal{P}$) for three values of the path voice loads ρ_p^v; recall that $r_p = 1$ for the radio network of Figure 2.

Similarly, Table 3 shows the average probability of data queueing $\bar{Q}$ (over all links) for different average voice loads $\bar{\rho}^v$, average data loads $\bar{\rho}^d$ defined as

$$\bar{\rho}^d = \frac{1}{|\mathcal{L}|} \sum_{l \in \mathcal{L}} \rho_l^d,$$

and for the three voice activity factor values 0.4, 0.8, and 1.0.

Then Tables 4a, 4b, and 4c show the individual probabilities of data queueing Q_l for each of the links l ($l \in \mathcal{L}$) for various data loads ρ_l^d, for voice activity factors 0.4, 0.8, and 1.0, and for voice loads 0.1, 1.0, and 2.0, respectively.

In the case of the single-rate radio network of Figure 2, since $r_p = 1$ for all $p \in \mathcal{P}$, if the utilizations ρ_p^v are equal for all paths, then $\bar{\rho}^v = \rho_p^v$ as well. Similarly, equal values of ρ_l^d for all data links imply that $\bar{\rho}^d = \rho_l^d$ as well. Therefore, since in all tables of this subsection the ρ_p^v's are equal to each other for all voice paths and the ρ_l^d's are equal to each other for all data links, no distinction is made in the text between the values of ρ_p^v and $\bar{\rho}^v$ or between the values of ρ_l^d and $\bar{\rho}^d$. In any event the caption of each table contains all necessary information about the voice and data loads.

In particular, in Table 1, the approximations and the exact value of average probability of voice blocking are compared for different values of the average offered voice load ($\bar{\rho}^v = \rho_p^v,\ p \in \mathcal{P}$). It is observed that the agreement between the approximations and the exact value is from excellent to very satisfactory over the entire range of values of the offered voice load considered.

We take the opportunity here to quantify the terms excellent, very satisfactory, and satisfactory (or fair) pertaining to approximation accuracy and used throughout this section. By excellent accuracy we mean that relative error between the approximation and the exact value is smaller than 1%; by very satisfactory we mean that the relative approximation error is smaller than 5%; and by satisfactory (or fair) we mean that the relative approximation error is most of the time smaller than 10% and occasionally between 10% and 20%. In all of our examples, the exact value always falls within the confidence interval provided by the Monte-Carlo summation method.

In Table 2, the results of Table 1 are shown in greater detail for the voice-blocking probabilities of each of the five voice circuits of the radio network and for offered voice loads equal to 1.0, 5.5, and 10.0 for all paths. Again, the approximations are very accurate for all voice circuits, although a variation in accuracy is observed from circuit to circuit.

Similarly, in Table 3, the knapsack approximation and the exact value of the average probability of data queueing are compared for different values of the offered voice and data load. For each of these tables the results are shown for data loads ($\bar{\rho}^d = \rho_l^d,\ l \in \mathcal{L}$) of 0.5, 2.5, and 4.0 (corresponding roughly to situations of light, moderate, and heavy data traffic) and for several values of the voice loads ($\bar{\rho}^v = \rho_p^v,\ p \in \mathcal{P}$). The accuracy of the knapsack approximation remains very satisfactory over the entire range of traffic parameters of interest.

Tables 4a - 4c show certain results of Table 3 in greater detail for the probabilities of data queueing at each of the nine data links of the radio network of Figure 2. The results are compared for the three voice activity factor values 0.4, 0.8, and 1.0 for voice loads 0.1 (Table 4a) and 1.0 (Table 4b) and data loads 0.5, 2.5, and 4.0. Results for percent error are included only for queueing probabilities greater than 0.0005. Again, the knapsack maintains from excellent to satisfactory accuracy over the entire range of traffic parameters of interest.

The results of Tables 2 and 3 (for probability of voice blocking) and Tables 3 and 4 (for probability of data queueing) demonstrate that the knapsack approximation is accurate not only when averages are taken over all circuits (paths) or links, but also when the network is examined at a more detailed "microscopic" level.

11.2.2 Results for a Network with Fewer Transceivers per Node

In the next group of tables (Tables 5, 6, and 7), we repeat the above results for the radio network of Figure 2 and a node transceiver vector $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$, that is, each node has four transceivers. The voice activity factor is $\beta/(\alpha + \beta) = 0.4$ for all results in this subsection. Table 5 shows the average probability of voice blocking (knapsack approximation and exact value) for different voice loads. It is observed that the accuracy of the knapsack approximation for this paradigm is inferior to that of the the radio network with transceiver vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$ (Tables 1 to 4); that is, there is a degradation in the accuracy of the approximation, as the number of the key network resource (the node transceivers) decreases. However, the (relative) approximation error still remains smaller than 5% of the exact value for offered voice load greater than 4.0.

Table 6 shows the knapsack approximation and the exact value of the average probability of data queueing for different voice and data loads. Similar trends to those observed for the average voice-blocking probability are observed here as well. Table 7 shows in detail certain results of Table 6 for the probabilities of data queueing of each of the nine data links of the radio network and, in particular, for the values 0.2, 1.0, and 2.0 of offered data load and the values 0.1 (in Table 7a), 1.0 (in Table 7b), 5.0 (in Table 7c), and 10.0 (in Table 7d) of the voice load. Again, the knapsack provides satisfactory performance, although the agreement with the exact values is not as good as for the larger system of Section 11.2.1.

11.2.3 Results for a System with Transceivers Reserved for Data

The last group of Tables in this subsection (Tables 8, 9, and 10) pertains to the radio network model of Figure 2 modified according to requirements of Section 7.3 so that finite data delays can be guaranteed. According to this model, a pair of transceivers for each of the nine data links is dedicated to data traffic; that is, the data link capacity vector $\underline{c}^d = (1,1,1,1,1,1,1,1,1)$ is guaranteed a priori. Thus, instead of the original node transceiver vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$, the five voice circuits of Figure 2b can only use the transceiver vector $\underline{T}^v = (5,7,7,7,4,7,4,7,7,7)$ that remains after the data transceivers have been assigned. The voice activity factor is $\beta/(\alpha+\beta) = 0.4$ for all results in this subsection.

In Table 8, the Monte-Carlo summation and knapsack approximations are compared to the exact value of the blocking probability for each of the five voice circuits and for the average value for a voice load of 2.5. The accuracy of the approximations remains satisfactory but it is inferior to that of the Tables 1 and 2 for which the voice transceiver vector was $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$; the smaller number of transceivers available for voice traffic degrades somewhat the accuracy of the approximations. In Tables 9 and 10, the average data delay

$$\bar{W} = \frac{\sum_{l\in\mathcal{L}} \rho_l^d W_l}{\sum_{l\in\mathcal{L}} \rho_l^d}$$

(measured in terms of packet duration prior to starting transmission) and the delays of each of the nine data links W_l ($l \in \mathcal{L}$), respectively, are shown in terms of the exact value and the knapsack approximation for values 2.5 and 10.0 of the offered voice load and 0.7, 0.9, and .999 of the offered data load. Again, as in the case of the probability of queueing data, the "exact value" for the queueing delay is actually a closed-form approximation based on the M/D/c model and Kleinrock's independence assumption. Satisfactory accuracy is observed in all cases.

11.3 Comparisons of Approximations for the Multi-Rate Network of Figure 3 without Admission Control

Figures 6 and 7 and Tables 11 to 14 pertain to performance results and comparisons of approximations for the multi-rate network of Figure 3, in the absence of admission control. The twelve voice circuits of this network (see Figure 3b) have rates $\underline{r}^v = (1,1,1,1,1,1,5,5,5,5,5,5)$ and the four data links have rates $\underline{r}^d = (1,1,1,1)$. Figures 6 and 7 pertain to the multi-rate network of Figures 3a and 3b with the link capacity vector $\underline{c} = (18,20,22,24)$. The voice activity factor is $\beta/(\alpha+\beta) = 0.4$ for all results in this subsection.

In Figure 6, the average voice blocking probability is depicted versus the offered voice load $\rho_p^v r_p$ over the range 0 to 3. The load $\rho_p^v r_p$ takes the same value for all twelve voice circuits. Whenever $\rho_p^v r_p$ ($p \in \mathcal{P}$) assumes a particular value, e.g., 1.5, the corresponding value of ρ_p is obtained from $1.5/r_p$ for each $p \in \mathcal{P}$, where r_p takes values 1 or 5 according to the vector of voice path (circuit) rates above.

For all tables in this subsection the offered loads (ρ_p^v) of the first six voice paths with $r_p = 1,\ p = 1, 2, \ldots, 6$ are equal to each other, and the offered loads of the remaining six voice paths with $r_p = 5,\ p = 7, 8, \ldots, 12$ are also equal to each other and equal to one fifth of the common value of the first six. This implies that the average voice load $\bar{\rho}^v = \rho_1^v = 5\rho_7^v$. Once these relationships are established the specific values of the loads are only shown in the caption of each table and they are not elaborated upon in the accompanying narrative.

As observed in Figure 6, the difference between the knapsack and Pascal approximations and the middle of the confidence interval of the Monte-Carlo summation is very small for the entire range of values of the offered voice load. Similarly, in Figure 7, the average probability of data queueing is illustrated as a function of the offered data load ρ_l^d. The data loads of all four links are assumed equal and the offered voice loads $\rho_p^v r_p$ are all equal to 2.0. Again excellent agreement is observed between the three approximations. The exact values for the probabilities of voice blocking and of data queueing are extremely time-consuming to compute for the multi-rate network in question, so they were not generated. However, since the Monte-Carlo summation method provides a confidence interval for the value of the performance measure of interest, we can judge the accuracy of the approximations even without having their exact values.

In Table 11, the average voice blocking probability is shown for the multi-rate network of Figure 2, different link capacity allocations, and different values of the offered voice loads. The four link capacity allocations considered are $\underline{c} = (90, 100, 110, 120)$, $\underline{c} = (18, 20, 22, 24)$, $\underline{c} = (9, 10, 11, 12)$, and $\underline{c} = (5, 5, 6, 6)$ and they are termed allocations 1, 2, 3, and 4, respectively. Allocation 1 is the example studied in [6]. The others are obtained by reducing the link capacities proportionally (or nearly so). The offered voice loads depicted in Table 11 have been selected so that they correspond to conditions of very light traffic (voice-blocking probability smaller than 0.1%), light to moderate traffic (voice-blocking probability around 1.0%), and moderate to heavy traffic (voice-blocking probability larger than 10.0%), for each of the four capacity allocation scenarios. Note that as the number of channels per link decreases, the offered voice load that corresponds to any particular blocking probability decreases at a much faster rate. For example, let us compare the results for capacity allocations 1 and 3. Although the link capacities are decreased by a factor of 10, the traffic levels corresponding to a specific blocking probability are decreased by a considerably larger factor, especially at low blocking probabilities. Such a behavior is expected; the availability of a large number of channels permits operation at higher throughput levels, because we can take advantage of the law of large numbers.

The knapsack and Pascal approximations remain relatively close to (but generally above) the upper edge of the confidence interval obtained via the Monte Carlo summation method. However, they appear to move farther away from that upper edge (corresponding to larger error) as the number of channels in the capacity allocation schemes decreases. Although we cannot claim that the knapsack approximation is uniformly more accurate than the Pascal approximation, it appears that the former ought to be trusted over a broader range of traffic (offered voice loads) and network (link capacities) parameters than the latter.

Tables 12a and 12b show in greater detail the voice-blocking probabilities of each of the twelve voice circuits of the network of Figure 3. In particular, in Table 12a the detailed results (knapsack, Pascal, and Monte Carlo approximations) are shown for offered voice loads of 8.0, 10.0, and 15.0 and for capacity allocation 1. The loads of the individual voice circuits are such that the average voice load takes the values 8.0, 10.0, or 15.0 once the rates of the different circuits have been accounted for. Again, the knapsack approximation appears to be superior to the Pascal approximation. Table 12b shows the probabilities of blocking for all twelve voice circuits for capacity allocation 2 and for offered voice loads 0.3, 0.7, and 1.7. The knapsack approximation appears to be superior to the Pascal approximation over a broad range of traffic and network parameters. The accuracy of the approximations exhibits a slight degradation, as the number of link channels in the capacity allocations decreases from that of capacity allocation 1 (90, 100, 110, 120) shown in Table 12a to that of capacity allocation 2 (18, 20, 22, 24) shown in Table 12b.

Tables 13 and 14 parallel the results of Tables 11 and 12 for the probability of data queueing. In particular, Table 13 shows the average probability of data queueing for the four capacity allocations and the different voice and data loads. The specific values of the offered voice and data loads have been selected so that they result in probabilities of data queueing of the order of 0.1%, 1.0%, and 10.0%, respectively, corresponding to light, moderate, and moderately heavy traffic. The knapsack and Pascal approximations appear to be either within or close to the edges of the confidence interval obtained via the Monte Carlo summation method. Again, the knapsack approximation appears to be superior to the Pascal approximation over a broad range of traffic and network parameters. Table 14 complements the results of Table 13 by showing the probabilities of data queueing at each of the four data links of the network of Figure 3 for capacity allocation 2 and for different voice and data loads. The voice loads, for which results are shown, are 0.3, 0.7, and 1.7; the data loads are selected in the manner discussed in Table 13. Observations similar to those made for Table 13 are valid here as well.

11.4 Threshold-Based Admission Control for the Radio Network of Figure 2

The group of Tables 15-20 presents a comparison of the voice blocking probabilities and probabilities of data queueing for the radio network of Figure 2, when admission control is used. In particular, three types of admission control are employed for deriving the results of Table 15: (i) Equal thresholds $X_p = 6$ on the voice traffic of all five circuits of Figure 2b, (ii) Optimal threshold-only policies such that $X_p \leq 6$ on the voice traffic of all five circuits, and (iii) Optimal full admission control policies that consist of thresholds X_p on individual circuit traffic and thresholds Y_s on linear combinations of circuit traffic over selected sets of circuits. The threshold constraints on individual circuit traffic take the form

$$n_p^v \leq X_p, \quad p = 1,2,3,4,5$$

The linear constraints on selected sets of voice circuits are

$$n_1^v + n_3^v \leq Y_1 \quad \text{(control constraints from node 5)}$$

$$n_1^v + n_4^v \leq Y_2 \quad \text{(control constraints from node 7)}$$

$$n_1^v + n_5^v \leq Y_3 \quad \text{(control constraints from node 5)}$$

$$n_3^v + n_5^v \leq Y_4 \quad \text{(control constraints from node 5)}$$

$$n_4^v + n_5^v \leq Y_5 \quad \text{(control constraints from node 7)}$$

These are identical to the admission control schemes considered in [11] and [12]. To the above control constraints one should add the resource constraints of the form

$$\sum_{p \in \mathcal{P}_n} n_p^v \leq T_n, \quad n \in \mathcal{N}$$

where T_n is the number of transceivers at node n and $\mathcal{P}_n$ is the set of paths intersecting at node n.

Together with the results of Table 15, one should also go back to Table 1, where no admission control was used. In Table 15, the optimal thresholds obtained for control policies (ii) and (iii) are shown for different values of the voice load; in the former case they are of the form $(X_1, X_2, X_3, X_4, X_5)$, in the latter case they are of the form $(X_1, X_2, X_3, X_4, X_5, Y_1, Y_2, Y_3, Y_4, Y_5)$. In this table, the optimal thresholds are selected on the basis of the exact average probability of voice blocking which is obtained from the product-form solution (as in [11]). The Monte Carlo summation and knapsack approximations are also shown for these thresholds, so that the accuracy of the approximations is checked against the exact value when admission control is used. The comparisons are favorable for both approximations, although the accuracy would be better if the number of transceivers at each node were higher. However, the knapsack approximation does not achieve its minimum value under the optimal thresholds of control policies (ii) and (iii); these optimal thresholds were obtained by using the exact value of the blocking probability. Consequently, we can not repeat the comparisons of [11] for the different control policies based on the knapsack approximation. However, as established with the results of Table 20 below, meaningful comparisons can be made for thresholds obtained from the knapsack approximation.

Table 16 shows the exact value and the knapsack approximation of the voice blocking probabilities for each of the five voice circuits of Figure 2b, voice loads $\rho_p^v = 2.5$ for all circuits, and several different thresholds of the admission control policy (ii). The knapsack approximation shows satisfactory accuracy for all situations examined. Table 17 presents similar results as Table 16, but for different thresholds of the full admission control policy (iii). Again, the knapsack approximation shows satisfactory accuracy for all situations examined, but the accuracy is inferior to that observed in Table 16. In both tables, the accuracy of the knapsack approximation can be improved at the expense of significant computational complexity, if the method of Section 9.2 is used instead of that of Section 9.3, which was used to derive the numerical results presented.

Tables 18 and 19 complement Tables 15, 16, and 17 by presenting results for the probabilities of data queueing in the presence of voice admission control. The voice activity factor is $\beta/(\alpha + \beta) = 0.4$ for all results in this subsection. Table 18 shows the average probability of data queueing for a data load of 2.5, for all nine data links and for different voice loads and admission control thresholds. The thresholds are the optimal thresholds for control policy (ii) in Table 15. Again, the observed accuracy of the knapsack approximation remains satisfactory, but inferior to that of networks without admission control (see Table 3). Similarly, Table 19 shows detailed results of the probabilities of data queueing at each of the nine data links of the network in question for a data load of 2.5 and different voice loads and admission control thresholds. Observations similar to those made for Table 18 are valid; moreover, the accuracy of the knapsack approximations appears to decrease as the offered voice load increases.

Table 20 compares the exact value and the knapsack approximation for the average voice blocking probability of the radio network of Figure 2 for different voice loads and three admission control scenarios: (a) no control, (b) threshold control on individual voice circuits based on thresholds selected by minimizing the knapsack-evaluated average blocking probability, and (c) threshold admission control based on optimal thresholds obtained by minimizing the exact average voice blocking probability [controls of type (ii) in Table 15]. The percentage improvement (decrease) of the exact blocking probability of the uncontrolled system when knapsack-based thresholds and optimal thresholds are used is also shown in separate columns.

As shown in Table 20, the optimal thresholds derived under policies (b) and (c) differ in one or more voice circuits; however, the performance obtained using the knapsack-optimized thresholds is almost as good as that obtained using the truly-optimal thresholds. For example for an offered voice load of 5.5, the optimal knapsack-optimized thresholds are (1,8,8,8,8) and incur an exact blocking probability of 0.453, whereas the truly optimal thresholds are (1,8,8,8,2) and incur an exact blocking probability of 0.448; the corresponding exact value for the uncontrolled system is 0.472. This corresponds to a 4% improvement (decrease) of the voice blocking probability when the knapsack-optimized thresholds are employed, versus the 5% decrease that can be achieved with the truly optimal thresholds. This improvement resulting from admission control is more drastic for large offered voice loads and for voice blocking probabilities. Up to a voice load of 6.5 the improvement increases as the voice load increases; however, for higher voice loads the improvement decreases with increasing offered voice load.

The results of Table 20 above together with those of Table 23 below (which establish the significant computational advantage of the knapsack approximation over the exact expressions) provide major motivation for applying the knapsack approximation to problems of control and optimization. Thus the knapsack approximation is useful not only for performance evaluation (as shown in Tables 1 to 14), but for control and optimization as well.

11.5 Voice and Data Revenue Sensitivities for Networks of Figures 2 and 3

The next group of tables (Tables 21a-21c and 22a-22c) pertains to the voice and data revenue sensitivities of the multi-rate network of Figure 3 and to the radio network of Figure 2. These sensitivities were derived in Section 10 and find diverse applications to problems of addition of network resources (e.g., additional transceivers), optimal data routing, joint voice control and data routing etc. In evaluating the revenue sensitivities we need the voice-revenue rates γ_p^v for all voice circuits, and the data revenue rates γ_l^d for all data links of the network. In Table 21, the revenue sensitivities are obtained for the network of Figure 3, an average voice load of 10.0 with circuit loads (10,10,10,10,10,10,2,2,2,2,2,2) and rates (1,1,1,1,1,1,5,5,5,5,5,5), and offered data loads of 50 for each of the data links in the network. The capacity allocation is $\underline{c} = (90, 100, 110, 120)$, the vector of voice revenue rates is arbitrarily chosen to be (1.0,1.2,1.4,1.6,1.8,2.0,3.0,3.6,4.2,4.8,5.4,6.0), and the vector of data revenue rates is (1.0,1.2,1.4,1.6). The voice activity factor is $\beta/(\alpha+\beta) = 0.4$ for all results in this subsection.

Table 21a shows the Monte-Carlo summation, knapsack, and Pascal approximations to the average voice revenue sensitivity with respect to the loads of all twelve voice circuits. The two approximations are either within or relatively close to the edges of the confidence interval obtained via the Monte Carlo summation method. Table 21b shows the average data revenue sensitivity with respect to the loads of each of the voice circuits. Again, the approximations and the confidence interval are in relatively satisfactory agreement with each other. Table 21c shows the average data revenue sensitivity with respect to the loads of the four data links of the network in question. Similar observations for the approximations like those for Table 21b are valid. It is significant that in all these tables the sign of the exact values of the sensitivities and of the approximate sensitivities is always the same. This guarantees that the the use of the approximate sensitivities (instead of the computationally inefficient exact expressions) in optimization routines will still move the optimization algorithm (e.g., the gradient descent algorithm) to the right direction for optimizing the objective function (performance measure) of interest.

In Table 22, the voice and data revenue sensitivities for the radio network of Figure 2 are presented for a vector of voice revenue rates (1.0,1.2,1.4,1.6,1.8) and for a vector of data revenue rates (1.0,1.2,1.4,1.6,1.8,2.0,2.2,2.4,2.6). In Table 22a, the exact value and the knapsack approximation to the average voice revenue sensitivity with respect to the loads of each of the five voice circuits are shown; the voice loads of all circuits are equal to 5.5. There is satisfactory agreement between the knapsack approximation and the exact value; the relative error remains below 10%, for most circuits. In Table 22b, the average data revenue sensitivity with respect to the loads of all five voice circuits of the same network are shown for a load of 5.5 for all voice circuits and for two different data load scenarios: one for which the load of all nine data links is equal to 4.0 and another for which it is 2.0. The accuracy of the knapsack approximation here is inferior to that observed in Table 22a, or in Table 21b. This is because the radio network has fewer resources and the knapsack approximation's accuracy decreases as the number of channels in each link decreases. In Table 22c, the average data revenue sensitivity with respect to the loads of each of the nine data links of the network in question is shown for a load of 5.5 on all five voice circuits and the two data load scenarios of Table 22b. The knapsack approximation here appears to be more accurate than that of Table 22b. Similar comments about the common sign of the approximate sensitivities and the exact expressions as for Tables 21a-21b are valid here.

Overall, the accuracy of the knapsack approximation for the voice and data revenue sensitivities appears reasonable and suggests that it can be used instead of the exact values (whose evaluation is computationally prohibitive) in problems of data routing, addition of resources, and so on. A detailed study of these issues will be addressed in the future.

11.6 Required Computational Effort for Various Approximations

Table 23 provides a comparison of the computational effort required for generating the various approximations and exact values of the probabilities of voice blocking and data queueing for the radio network of Figure 2 and the multi-rate network of Figure 3. In the framework of this comparison, the radio network of Figure 2 is typical of a small-size network and that of Figure 3 is typical of a medium- to large-size network. SUN SPARC Station II workstations were employed for all necessary computations. The computer code was written in C and was not optimized.

From the table we see that the computational advantage of the knapsack and Pascal approximations over the exact expression is tenfold and over the Monte Carlo summation is thirtyfold for the small-size network of Figure 2. For the mid-size network of Figure 3, the advantage in speed of the knapsack and Pascal approximations over the Monte Carlo summation becomes six-hundred-fold, whereas the time necessary for the evaluation of the exact value is prohibitive for most computers. With the latter statement we mean that the required computation time is estimated to be of the order of 6 to 7 days for that paradigm and thus useless for optimization (control or resource allocation) purposes.

The computation time for the probabilities of voice blocking and data queueing appears to be identical for both network paradigms, despite the fact that the probability of data queueing is inherently more complicated (additional conditioning and averaging are involved), for the following reason. We store the pre-computed (recursively) values of the conditional M/D/c data queueing probability (conditioned on the voice state) and then use them in the knapsack approximation (refer to Section 5 for the appropriate equations indicating the conditioning and final averaging). Of course, the memory (storage) requirements for the evaluation of the probability of data queueing are considerably larger than those of the probability of voice blocking.

Table 23 establishes the unquestionable computational advantage of the knapsack and Pascal approximations over the exact expressions for the performance measures of interest and for networks of any size. These two approximations appear to require a computational effort that remains relatively insensitive to the network size. The Monte Carlo summation method is computationally efficient only for small- and mid-size networks. These facts, together with the very satisfactory accuracy of the approximations over the entire range of traffic and network parameters of interest, establish these approximations as excellent candidates for the time-efficient and accurate performance evaluation of networks of arbitrary size and resources (link capacities), as well as for the derivation of near-optimal control schemes that optimize the above performance measures.

12. CONCLUSIONS

In this section we recapitulate the most important features of the approximations developed in this report and draw conclusions about their applicability to problems of network control and optimization, sensitivity analysis, and performance evaluation.

12.1 Accuracy and Computational Advantage of Approximations

In this report, we developed reduced-load approximations (based on the stochastic knapsack and the Pascal distribution) and Monte-Carlo summation techniques, which enable the accurate and computationally efficient evaluation of the probability of blocking of voice calls, the probability of data link queueing, and the average data link delay in integrated multi-hop radio networks. Accurate and time-efficient approximations are necessary, because the computational effort for the evaluation of the exact expressions for these performance measures is prohibitive for networks of even moderate size. By contrast, the approximations of this report (refer to Table 23) are very time-efficient.

12.2 Application of New Approximation Techniques to a Broad Range of Network Architectures and Traffic Types

Although the knapsack, Pascal, and Monte-Carlo Summation methods had already been applied to multi-rate voice-only circuit-switched networks, they had never been applied before either to the performance evaluation of multi-hop radio networks or to networks with integrated voice and data traffic. And, even for multi-rate voice networks, they were thought to be accurate only in the limiting regime characterized by large link capacities and large offered voice loads while their ratio maintained a fixed value.

In this report, besides extending the applicability of the aforementioned approximations to **radio networks** (with transceivers at the nodes being the network resource rather than the link capacities) and to **integrated voice and data traffic**, we showed that these approximations exhibit excellent to very satisfactory accuracy, for the **entire range of network parameters** (e.g., size, link capacities, number of transceivers at nodes) and traffic parameters (e.g., voice and data loads, voice activity and silence periods) of interest. Moreover, the accuracy of the approximations is very satisfactory not only for the **average** performance measures (i.e., averaged over all circuits and links of the network) but also for the performance measures characterizing **individual** circuits or links.

These approximation methods are applicable to **single-rate networks** characterized by a single bandwidth assignment for all voice circuits, as well as to **multi-rate networks**, in which calls with different bandwidth requirements are accepted; in both cases, the data may be transmitted at a lower rate than that of the voice traffic. Moreover, our methods can handle **voice models with periods of activity and silence,** in which data is transmitted during the silent periods of ongoing voice calls. Finally, our methods are applicable to **variable-rate** traffic **voice or video**; in these situations the rate of the source can assume a number of different values (from a fixed set) as time varies. Our preliminary results actually show that our approach (based on the knapsack approximation) is applicable to sophisticated **Markov-Modulated Poisson Process (MMPP)** models for the variable-rate voice or video traffic with only a small increase of computational complexity. In this context, besides variable-rate voice, **video telephony** as well as **full-motion video** sources can be modeled and networks with such traffic can be analyzed.

12.3 Application to Control and Optimization Problems

Besides employing these approximation methods for the **accurate and time-efficient performance evaluation** of integrated multi-hop radio networks (and of more general multi-rate networks) we can use them for **control and optimization** purposes, because of their computational efficiency. The recommended general methodology here is to use an accurate and computationally time-efficient approximation (e.g., knapsack) to evaluate the specific performance measure(s) (single measure or multiple measures in a weighted sum) of interest and carry out the minimization (optimization in general) with respect to the most promising classes of controls. The controls derived will be near-optimal in the sense that their performance will be very close to those of the really optimal ones that are derived from the optimization of the exact performance measures.

In particular, we have shown (refer to Table 20) that **threshold policies for voice-admission control** that minimize the knapsack-evaluated probability of voice blocking improve substantially the performance of the controlled network over that of the network without admission control and actually come very close to the performance of the optimal thresholds (which minimize the actual voice blocking probability). Without these computationally efficient accurate approximations to the performance measures of interest, the derivation of near-optimal control schemes is not feasible and one has to resort to ad-hoc designs.

Moreover, the **sensitivities** of suitably defined **voice and data revenue measures with respect to link capacities (or number of node transceivers), voice loads, and data loads** were evaluated via the knapsack approximation and shown to be very close to the actual values (refer to Tables 21 and 22). Again, these approximate sensitivities are much more computationally efficient than the cumbersome (and usually prohibitive) exact expressions. Actually, as our results establish, there will be almost negligible loss in revenue when these approximate sensitivities are used in place of the exact ones. Consequently, important practical problems of **allocating additional network resources** in response to increasing voice and/or data network traffic demand can be easily handled with our approach, as well as problems of **data routing** in which the derivatives of the data delay (or the probability of queueing) are used by standard optimal routing algorithms.

12.4 Application to High-Speed Networks

The approximations to the probabilities of voice blocking, data queueing, and the average data delay developed in this report can be also applied to important problems in the area of **high-speed networks**, with proper modification. These include:

(i) algorithms for the set up of virtual paths for video, voice, and high rate data sources in asynchronous transfer mode (ATM) networks.

(ii) schemes for admission control of different classes of traffic in ATM networks, and

(iii) specific formulations of the problem of multicasting hierarchically encoded data to destinations that can receive subsets of the transmitted signal according to their bandwidth and access constraints.

In all such high-speed network applications, the approximations of this report are expected (after suitable modification) to provide both accurate and time-efficient performance evaluations and to be used for deriving near-optimal control schemes and resource allocations.

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APPENDIX A

Proof of Part (a) of Proposition 3.1

(System Steady-State Distribution)

The starting point is the chain rule for the steady-state probability of the voice state $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$, namely

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}, n_l^d) = P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot P(n_l^d | \underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}). \qquad (A-1)$$

Since voice has preemptive priority over data, the analysis of the voice component can be isolated from that of the data component. Consider the voice state vector $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$; denote

$$\underline{N}^{v+}_{\mathcal{P}p} = (n_1^v, \ldots, n_{p-1}^v, n_p^v + 1, n_{p+1}^v, \ldots, n_{|\mathcal{P}|}^v)$$

$$\underline{N}^{v-}_{\mathcal{P}p} = (n_1^v, \ldots, n_{p-1}^v, n_p^v - 1, n_{p+1}^v, \ldots, n_{|\mathcal{P}|}^v)$$

$$\underline{N}^{s+}_{\mathcal{P}p} = (n_1^s, \ldots, n_{p-1}^s, n_p^s + 1, n_{p+1}^s, \ldots, n_{|\mathcal{P}|}^s)$$

$$\underline{N}^{s-}_{\mathcal{P}p} = (n_1^s, \ldots, n_{p-1}^s, n_p^s - 1, n_{p+1}^s, \ldots, n_{|\mathcal{P}|}^s).$$

The global balance equation for $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$ is

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot \sum_{p \in \mathcal{P}} \left[n_p^s \mu_p^v + (n_p^v - n_p^s)\mu_p^v + (n_p^v - n_p^s)\alpha + n_p^s \beta + F_p^v \frac{\alpha}{\alpha + \beta} + F_p^v \frac{\beta}{\alpha - \beta} \right] =$$

$$= \sum_{p \in \mathcal{P}} \Big[P(\underline{N}^{v-}_{\mathcal{P}p}, \underline{N}^{s-}_{\mathcal{P}p}) \cdot F_p^v \frac{\alpha}{\alpha + \beta} + P(\underline{N}^{v-}_{\mathcal{P}p}, \underline{N}^s_{\mathcal{P}}) \cdot F_p^v \frac{\beta}{\alpha + \beta}$$

$$+ P(\underline{N}^v_{\mathcal{P}}, \underline{N}^{s+}_{\mathcal{P}p}) \cdot (n_p^s + 1)\beta + P(\underline{N}^v_{\mathcal{P}}, \underline{N}^{s-}_{\mathcal{P}p}) \cdot (n_p^v - n_p^s + 1)\alpha$$

$$+ P(\underline{N}^{v+}_{\mathcal{P}p}, \underline{N}^{s+}_{\mathcal{P}p}) \cdot (n_p^s + 1)\mu_p^v + P(\underline{N}^{v+}_{\mathcal{P}p}, \underline{N}^s_{\mathcal{P}}) \cdot (n_p^v - n_p^s + 1)\mu_p^v \Big] \qquad (A-2)$$

for all $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}), (\underline{N}^{v\pm}_{\mathcal{P}p}, \underline{N}^{s\pm}_{\mathcal{P}p}) \in \Omega_s$.

The local balance equations for $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$ are

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot n_p^s \mu_p^v = P(\underline{N}^{v-}_{\mathcal{P}p}, \underline{N}^{s-}_{\mathcal{P}p}) \cdot F_p^v \frac{\alpha}{\alpha + \beta} \qquad (A-3)$$

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot (n_p^v - n_p^s)\mu_p^v = P(\underline{N}^{v-}_{\mathcal{P}p}, \underline{N}^s_{\mathcal{P}}) \cdot F_p^v \frac{\beta}{\alpha + \beta} \qquad (A-4)$$

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot (n^v_p - n^s_p)\alpha = P(\underline{N}^v_{\mathcal{P}}, \underline{N}^{s+}_{\mathcal{P}p}) \cdot (n^s_p + 1)\beta \qquad (A-5)$$

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot n^s_p\beta = P(\underline{N}^v_{\mathcal{P}}, \underline{N}^{s-}_{\mathcal{P}p})(n^v_p - n^s_p + 1)\alpha \qquad (A-6)$$

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot F^v_p \frac{\alpha}{\alpha+\beta} = P(\underline{N}^{v+}_{\mathcal{P}p}, \underline{N}^{s+}_{\mathcal{P}p}) \cdot (n^s_p + 1)\mu^v_p \qquad (A-7)$$

$$P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}) \cdot F^v_p \frac{\beta}{\alpha+\beta} = P(\underline{N}^{v+}_{\mathcal{P}p}, \underline{N}^s_{\mathcal{P}}) \cdot (n^v_p + 1 - n^s_p)\mu^v_p \qquad (A-8)$$

for all $(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}}), (\underline{N}^{v\pm}_{\mathcal{P}p}, \underline{N}^{s\pm}_{\mathcal{P}p}) \in \Omega_s$.

It is straightforward to show that the $P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$ provided in Part (a) of Proposition 1 is a solution to the above local balance equations. Any solution to the local balance equation must also be a solution to the global balance equation and, as $P(\underline{N}^v_{\mathcal{P}}, \underline{N}^s_{\mathcal{P}})$ in Part (a) satisfies the balance equation, it is the steady state probability.

APPENDIX B

M/M/c and M/D/c Data Models

This appendix reviews all important formulas of the M/M/c and M/D/c data models and sketches the relaxation method proposed by Tijms [13] for the evaluation of the steady state probabilities and other quantities of interest for the M/D/c model.

B.1 Useful Expressions for M/M/c Data Model

The steady-state probabilities satisfy the recursion

$$\lambda p_{j-1} = \min(j, c)\mu p_j, \qquad j = 1, 2, \ldots$$

and are given by the expressions

$$p_j = \begin{cases} \frac{(c\rho)^j}{j!} p_0, & j = 0, 1, \ldots, c-1, \\ \frac{(c\rho)^j}{c! c^{j-c}} p_0, & j \geq c \end{cases}$$

where

$$p_0 = \left\{ \sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)} \right\}^{-1}.$$

The probability of finding the system busy (blocking probability) $\Pi_w = P\{j \geq c\} = \sum_{j=c}^{\infty} p_j$ is given by

$$\Pi_w = \frac{(c\rho)^c}{c!(1-\rho)} \left\{ \sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)} \right\}^{-1}.$$

The average number of customers in the queue $E\{N_q\} = \sum_{j=c}^{\infty} (j-c) p_j$ is given by

$$E(N_q) = \frac{(c\rho)^c \rho}{c!(1-\rho)^2} \left\{ \sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)} \right\}^{-1}.$$

Finally, the probability distribution of queueing delay (i.e., waiting time, not including service time) W_q is given by

$$P\{W_q > x\} = \sum_{j=c}^{\infty} p_j \sum_{k=0}^{j-c} e^{-c\mu x} \frac{(c\mu x)^k}{k!} = \Pi_w e^{-c\mu(1-\rho)x}, \quad x \geq 0.$$

and the average queueing delay is given by

$$E(W_q) = \frac{E(N_q)}{\lambda} = \frac{(c\rho)^c}{c!c\mu(1-\rho)^2}\left\{\sum_{k=0}^{c-1}\frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)}\right\}.$$

B.2 Useful Expressions for M/D/c Data Model

The steady-state probabilities satisfy the following equilibrium equations

$$p_0(t+D) = \sum_{k=0}^{c} p_k(t)e^{-\lambda D}$$

and

$$p_j(t+D) = \sum_{k=0}^{c} p_k(t)e^{-\lambda D}\frac{(\lambda D)^j}{j!} + \sum_{k=c+1}^{c+j} p_k(t)e^{-\lambda D}\frac{(\lambda D)^{j-k+c}}{(j-k+c)!}, \quad j \geq 1$$

where D is the service time.

From those we can derive the recursion

$$p_j = e^{-\lambda D}\frac{(\lambda D)^j}{j!}\sum_{k=0}^{c} p_k + \sum_{k=c+1}^{c+j} p_k e^{-\lambda D}\frac{(\lambda D)^{j-k+c}}{(j-k+c)!}, \quad j = 0, 1, \ldots$$

where

$$\sum_{j=0}^{\infty} p_j = 1.$$

The average number of customers in the queue can be also derived as

$$E(N_q) = \frac{(c\rho)^2 - c(c-1) + \sum_{j=0}^{c-1}\{c(c-1) - j(j-1)\}p_j}{2c(1-\rho)}$$

and its second moment as

$$E(N_q(N_q-1)) = \frac{(c\rho)^3 - c(c-1)(c-2) + \sum_{j=0}^{c-1}\{c(c-1)(c-2) - j(j-1)(j-2)\}p_j}{3c(1-\rho)}$$

$$- \frac{(c-1-c\rho^2)}{1-\rho}E(N_q).$$

B.3 Relaxation Algorithm for Evaluating Steady State Probabilities of M/D/c Data Model

The above recursion for the p_j's can not be used directly to find the p_j's of the M/D/c system because it is equivalent to an infinite system of linear equations. Thus we first truncate it to a finite system by using a sufficiently large integer L chosen so that

$$\sum_{j=L}^{\infty} p_j \leq \sum_{j=L}^{\infty} p_j^{\text{exp}} \leq 10^{-9}$$

where p_j^{exp} for $j = 0, 1, \ldots$ denote the steady state probabilities of the M/M/c system, which are available in closed form (see Section B.1 above); the inequality reflects the intuitive fact that the M/D/c queue involves less variability than the M/M/c queue.

It turns out (see [13]) that for light traffic it is preferable to use the truncation to L terms described above. However, for non-light traffic, unacceptably large values of L may be needed. In this case it is advantageous to use the theoretical fact [13] that

$$\frac{p_{j-1}}{p_j} \approx \tau \quad \text{for all sufficiently large j}$$

where

$$\tau = 1 + \delta/\lambda$$

and δ is determined as the unique positive solution to the equation

$$\lambda(e^{\delta D/c} - 1) = \delta$$

This asymptotic result is exploited in the following manner: for all j such that $0 \leq j \leq N$ we use p_j and try to determine their value from the system of linear equations; for all $j > N$ we use

$$p_j \approx \tau^{N-j} p_N \quad \text{for} \quad j > N$$

where $N > c$. It turns out (see [13]) that the necessary value of N is smaller than the necessary value of L (of the previous paragraph). The procedure for obtaining the p_j's is sketched next (following [13]).

After truncating the linear system of equations involving the p_j's to an $(N+1)\times(N+1)$ linear system and replacing the probabilities p_j by $p_N\tau^{N-j}$, for $j \geq N$, we obtain the following system of linear equations:

$$p_j = \sum_{\substack{k=0 \\ k\neq j}}^{N} a_{jk} p_k, \qquad j = 0, 1, \ldots, N$$

with

$$\sum_{j=0}^{N} p_j + \frac{\tau}{\tau - 1} p_N = 1 \tag{16}$$

where

$$a_{jk} = \begin{cases} \frac{a(\min(j,j-k+c))}{1-a(\min(j,c))}; & 0 \leq j \leq N-1, \ \ 0 \leq k \leq \min(c+j, N-1) \\ \frac{\sum_{k=N}^{c+j} \tau^{N-k} a(j-k+c)}{1-a(\min(j,c))}; & N-c \leq j \leq N-1, \ \ k = N \\ \frac{a(\min(j,j-k+c))}{1-\sum_{k=N}^{c+j} \tau^{N-k} a(j-k+c)}; & j = N, \ \ 0 \leq k \leq N-1 \\ 0; & \text{otherwise} \end{cases}$$

Here,

$$a(\ell) = e^{-\lambda D}(\lambda D)^{\ell}/\ell!, \quad \text{for } \ell \geq 0.$$

The above system of linear equations is solved via the modified successive overrelaxation method of Tijms [13].

According to the standard successive overrelaxation method the operator B_ω associated with a relaxation factor ω transforms each vector $x = (x_0, x_1, \ldots, x_N)$ into the vector $B_\omega x$, whose components $(B_\omega x)_i$ are defined recursively by

$$(B_\omega x)_i = (1-\omega)x_i + \omega \left[\sum_{j=0}^{i-1} a_{ij}(B_\omega x)_j + \sum_{j=i+1}^{N} a_{ij} x_j \right] \quad i = 0, 1, \ldots, N.$$

Assuming that the integer N is sufficiently large—so that the reduced system of linear equations has a solution—then this solution is an eigenvector of B_ω with associated eigenvalue

1. Letting $\lambda_1(\omega)$ be the eigenvalue having the largest absolute value among the eigenvalues of B_ω unequal to 1, the standard successive overrelaxation method with a fixed relaxation factor ω converges only if $|\lambda_1(\omega)| < 1$. Moreover, the standard overrelaxation method has the best convergence rate for that value of ω for which $|\lambda_1(\omega)| < 1$ is smallest. It should be noted that the optimal value of ω may be rather sensitive to the parameters of the specific problem considered and, in some cases, will be close to 1.

The problems of the standard overrelaxation method are avoided with Tijms' algorithm [13]. In this algorithm ω is always kept between 1 and 2. In case $\lambda_1(\omega)$ is real, the algorithm estimates $\lambda_1(\omega)$ after some iteration of the overrelaxation method (see the parameter r^h in the algorithm below). This estimate provides a method to formulate a successive overrelaxation algorithm, in which the relaxation factor is dynamically adjusted in order to search for that value of ω, for which $|\lambda_1(\omega)| < 1$ is smallest. The steps of Tijms' algorithm for the calculation of the state probabilities in the M/D/c queue are provided below.

Special-Purpose Overrelaxation Method for the M/D/c Queue

Step 0. Choose $N > c$ and $x^0 \geq 0$ with

$$\sum_{i=0}^{N} x_i^0 + \tau(\tau-1)^{-1} x_N^0 = 1$$

Also, $h := 0$ and $\omega := 1.20$.

Step 1. $\omega^{old} := 0$, $\lambda(\omega^{old}) := 1$, $f^h := r^h := \infty$.

Step 2. $h := h + 1$. Compute the vectors

$$\tilde{x}^h = B_\omega x^{h-1}$$

$$h_i^h = \left[\sum_{i=0}^{N} |\tilde{x}_i^h| + \frac{\tau}{\tau-1}|\tilde{x}_N^h|\right]^{-1} \tilde{x}^h$$

and the scalar

$$f^h := \frac{1}{N}\sum_{i=0}^{N}\left|\frac{\tilde{x}_i^h - \tilde{x}_i^{h-1}}{\tilde{x}_i^h}\right|$$

If $f^h < \epsilon_N$, with ϵ_N a prespecified accuracy number, then go to Step 4. Otherwise

$$r^h := \frac{f^h}{f^{h-1}}$$

If $r^h \geq 1$ or $h \geq 10$, then ω is likely too large and decrease ω, as $\omega := 1 + \frac{1}{2}(\omega - 1)$, put $x^0 := x^h$ and $h := 0$, and go to Step 1. If $r^h < 1$ and r^h has sufficiently converged, according to $|(r^h - r^{h-1})/r^h| < 0.025$, then go to step 3; otherwise, return to Step 2.

Step 3. $\lambda(\omega) := r^h$. Test for one of the following four possibilities: (a) $\omega > \omega^{old}$ and $\lambda(\omega) > \lambda(\omega^{old})$; (b) $\omega > \omega^{old}$ and $\lambda(\omega) \leq \lambda(\omega^{old})$; (c) $\omega < \omega^{old}$ and $\lambda(\omega) > \lambda(\omega^{old})$; (d) $\omega < \omega^{old}$ and $\lambda(\omega) \leq \lambda(\omega^{old})$. For the cases (a) and (d),

$$\omega^{old} := \omega, \quad \lambda(\omega^{old}) := \lambda(\omega), \quad \omega := 1 + 0.85(\omega - 1)$$

whereas, for the cases (b) and (c),

$$\omega^{old} := -\omega, \quad \lambda(\omega^{old}) := \lambda(\omega), \quad \omega := 1 + 1.25(\omega - 1)$$

Next, $x^0 := x^h$, $h := 0$, $f^h := r^h := \infty$, and then go to Step 2.

Step 4. If

$$|\sum_{i=0}^{N} \tilde{x}_i^h + \frac{\tau}{\tau - 1}\tilde{x}_h^N - 1| < 10^{-5}$$

and

$$\left|\sum_{i=0}^{c-1} i x_i^h + c\left(1 - \sum_{i=0}^{c-1} x_i^h\right) - c\rho\right| < 10^{-5}$$

then the algorithm is stopped and the state probabilities p_i are obtained from

$$p_i = x_i^h \text{ for } 0 \leq i \leq N \text{ and } p_i = \tau^{N-1} x_N^h \text{ for } i > N.$$

The above stopping criteria use the fact that the probabilities sum to 1 and that the average number of busy servers equals $c\rho$. Otherwise,

$$x_i^0 := x_i^h \text{ for } 0 \leq i \leq N \text{ and } x_i^0 := \frac{1}{\tau} x_{i-1} \text{ for } N \leq i \leq N + 10$$

$N := N + 10$, $h := 0$, and then go to Step 1.

B.4 Recursive Algorithm for Evaluating the Probability Distribution of Queueing Delay under the M/D/c Data Model

To compute the probability distribution of the queueing delay W_q, that is $P\{W_q \leq x\}$, we first find an integer m and the remainder u $(0 \leq u < D)$ such that

$$x = mD + u.$$

Then we obtain the desired quantity from

$$P\{W_q \leq x\} = b_{mc+c-1}(u)$$

where the $b_l(u)$ satisfy the recursion

$$\sum_{i=0}^{j} p_i = \sum_{k=0}^{j} b_{j-k}(u) e^{-\lambda u} \frac{(\lambda u)^k}{k!}, \quad \text{for} \quad j = 0, 1, \ldots$$

with initial value

$$b_0(u) = e^{\lambda u} p_0.$$

Actually, for computational purposes the following recursion is preferred

$$b_j(u) = e^{\lambda u} \sum_{i=0}^{j} p_i - \sum_{k=0}^{j-1} b_k(u) \frac{(-\lambda u)^{j-k}}{(j-k)!}, \quad \text{for} \quad j = 1, 2, \ldots$$

or equivalently

$$b_j(u) = e^{\lambda u} p_j - \sum_{k=0}^{j-1} b_k(u) \frac{(-\lambda u)^{j-1-k}}{(j-1-k)!} \left(\frac{\lambda u}{j-k} - 1 \right), \quad \text{for} \quad j = 1, 2, \ldots$$

The above recursion should be used in combination with the approximation

$$P\{W_q > x\} \approx \alpha e^{-\delta x} \text{ for } x \geq D/\sqrt{c}.$$

In the above approximation δ is again determined as the solution to the equation

$$\lambda(e^{\delta D/c} - 1) = \delta$$

and α is given by

$$\alpha = \frac{\delta\eta}{\lambda(\tau-1)^2\tau^{c-1}}$$

where

$$\tau = 1 + \frac{\delta}{\lambda}$$

and

$$\eta = \lim_{j\to\infty} \tau^j p_j = \frac{\sum_{i=0}^{c-1} p_i(\tau^{i-1} - \tau^{c-1})}{c/\tau - \lambda D}$$

APPENDIX C

Proof of Proposition 5.1

(Knapsack Approximation)

From the proof of Proposition 3.1 we know that $P(\underline{N}^v, \underline{N}^s)$ satisfies several local balance equations. But actually the first two of them are sufficient for solving $P(\underline{N}^v, \underline{N}^s)$. For the same reason, the following two local balance equations are sufficient for obtaining $P(k, m)$

$$\frac{\alpha}{\alpha+\beta}\rho_p^v P(\underline{N}_p^{v-}, \underline{N}_p^{s-}) = n_p^s P(\underline{N}^v, \underline{N}^s), \quad n_p^v,\ n_p^s \geq 1 \qquad (C-1)$$

$$\frac{\beta}{\alpha+\beta}\rho_p^v P(\underline{N}_p^{v-}, \underline{N}_p^{s}) = (n_p^v - n_p^s) P(\underline{N}^v, \underline{N}^s), \quad n_p^v \geq 1 \qquad (C-2)$$

where we have dropped the subscript $\mathcal{P}$ in $\underline{N}_{\mathcal{P}p}^{v-}$, $\underline{N}_{\mathcal{P}p}^{s-}$, and in the other vectors in order to simplify the notation; as usual, $\rho_p^v = F_p/\mu_p^v$. Summing both sides of equation (C-1) with respect to $(\underline{N}^v, \underline{N}^s) \in \Omega(k, m)$, one can obtain

$$\frac{\alpha}{\alpha+\beta}\rho_p^v \sum_{(\underline{N}^v, \underline{N}^s)\in\Omega(k,m)\cap\{n_p^v, n_p^s \geq 1\}} P(\underline{N}^{v-}, \underline{N}^{s-}) = \sum_{(\underline{N}^v, \underline{N}^s)\in\Omega(k,m)} n_p^s P(\underline{N}^v, \underline{N}^s) \quad (C-3)$$

where

$$\Omega(k,m) \cap \{n_p^v, n_p^s \geq 1\} =$$

$$= \Big\{(\underline{N}^v, \underline{N}^s | 1 \leq n_p^s \leq n_p^v,\ r_p \leq r_p n_p^v \leq c_l,\ 0 \leq n_q^s \leq n_q^v,\ 0 \leq r_q n_q^v \leq c_l,\ q, p \in \mathcal{P}_l, q \neq p;$$

$$\left.\sum_{q\in\mathcal{P}_l, q\neq p} r_q n_q^s + r_p(n_p^s - 1) = m - r_p,\ \sum_{q\in\mathcal{P}_l, q\neq p} r_q n_q^v + r_p(n_p^v - 1) = k - r_p \right\}.$$

For the RHS of (C-3) we used the fact that, if $n_p^s = 0$ then RHS $= 0$ and if $n_p^v = 0$ then $n_p^s = 0$ and thus RHS $= 0$. If we define

$$\hat{n}_q^s = \begin{cases} n_p^s - 1, & p = q \\ n_q^s, & p \neq q \end{cases}$$

$$\hat{n}_q^v = \begin{cases} n_p^v - 1, & p = q \\ n_q^v, & p \neq q \end{cases}$$

and replace (n_q^v, n_q^s) by $(\hat{n}_q^v, \hat{n}_q^s)$ in the LHS of (C-3), then $\Omega(k,m) \cap \{n_p^v, n_p^s \geq 1\} = \Omega(k - r_p, m - r_p)$ with the new variables and we can rewrite the LHS of (C-3)

$$\frac{\alpha}{\alpha+\beta}\rho_p^v P(k - r_p, m - r_p).$$

Moreover, we can rewrite the RHS of (C-3) as

$$\sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k,m)} n_p^s \frac{P(\underline{N}^v, \underline{N}^s)}{P(k,m)} P(k,m).$$

Note that

$$P\left(\underline{N}^v, \underline{N}^s \middle| \sum_{q \in \mathcal{P}_l} r_q n_q^s = m, \sum_{q \in \mathcal{P}_l} r_q n_q^v = k\right) = \begin{cases} \frac{P(\underline{N}^v, \underline{N}^s)}{P(k,m)}, & (\underline{N}^v, \underline{N}^s) \in \Omega(k,m) \\ 0, & \text{otherwise} \end{cases}.$$

Denote

$$E[n_p^s | k, m] = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k,m)} n_p^s \cdot P\left(\underline{N}^v, \underline{N}^s \middle| \sum_{q \in \mathcal{P}_l} r_q n_q^s = m, \sum_{q \in \mathcal{P}_l} r_q n_q^v = k\right).$$

Then the RHS of (C-3) becomes

$$E[n_p^s | k, m] \cdot P(k,m) \quad .$$

Equating the modified LHS and RHS of (C-3) we have

$$\frac{\alpha}{\alpha+\beta}\rho_p^v P(k - r_p, m - r_p) = E[n_p^s | k, m] \cdot P(k,m). \qquad (C-4)$$

Following similar arguments, we obtain

$$\frac{\beta}{\alpha+\beta}\rho_p^v P(k - r_p, m) = E[n_p^v - n_p^s | k, m] \cdot P(k,m) \qquad (C-5)$$

from equation (C-2). Multiplying equations (C-4) and (C-5) by r_p, summing with respect to $p \in \mathcal{P}_l$, and using the facts that

$$E\left\{ \sum_{p \in \mathcal{P}_l} r_p n_p^s \middle| k, m \right\} = m$$

and

$$E\left\{\sum_{p\in\mathcal{P}_l} r_p\left(n_p^v - n_p^s\right)\middle| k, m\right\} = k - m$$

we obtain

$$\frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v P(k-r_p, m-r_p) = m\cdot P(k,m)\ , \quad r_p \le m \le k \le c_l$$

and

$$\frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v P(k-r_p, m) = (k-m)\cdot P(k,m), \quad 0 = m < r_p \le k \le c_l.$$

Finally, by defining

$$P'(k,m) = P(k,m)/P(0,0)$$

we derive (5.10a) and Proposition 5.1 follows.

APPENDIX D

Four-Dimensional Knapsack Approximation

(Proofs of Propositions 5.2 and 9.1)

In this appendix we provide the recursion for the computation of the four-dimensional knapsack approximation used for evaluating the probability of queueing of data and the average queueing delay of data (Sections 5.2 and 7.2) and for the probability of voice blocking with admission control (Section 9). We present the proof only for Proposition 5.2; Proposition 9.1 follows as a special case of 5.2.

Here we provide a recursion for $P(k_1, m_1, k_2, m_2)$ defined as

$$P(k_1, m_1, k_2, m_2) = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k_1, m_1, k_2, m_2)} P(\underline{N}^v, \underline{N}^s)$$

where $\mathcal{P}_i$ for $i = 1, 2$ are two arbitrary sets of paths, with $\mathcal{P}_1 \neq \mathcal{P}_2$.

$$0 \leq k_i = \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^v \leq Z_i, \qquad i = 1, 2$$

$$0 \leq m_i = \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^s \leq k_i, \qquad i = 1, 2.$$

Notice that these inequalities can represent any constraint involving a linear combination of the number of voice calls n_p^v of all classes $p \in \mathcal{P}_i$. The coefficients $r_p^{(i)}$ $(i = 1, 2)$ may be equal to the rates r_p or may be arbitrary non-negative constants. For example, $Z_1 = c_l$ and $r_p^{(1)} = r_p$ whereas $Z_2 = T_s$ and $r_p^{(2)} = r_p^{(s)}$ corresponds to the scenario of Section 9.1. Also the general case of unequal rates $r_p^{(i)}$ of path p under the two constraints $i = 1, 2$ (i.e., $r_p^{(1)} \neq r_p^{(2)}$) can be handled by our analysis. Finally, $\Omega(k_1, m_1, k_2, m_2)$ is defined by

$$\Omega(k_1, m_1, k_2, m_2) = \Big\{ (\underline{N}^v, \underline{N}^s) | 0 \leq n_p^s \leq n_p^v, p \in \mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2;\ \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^v = k_i,$$

$$\sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^s = m_i, i = 1, 2 \Big\}$$

$$\sum_{k_1=0}^{Z_1}\sum_{m_1=0}^{k_1}\sum_{k_2=0}^{Z_2}\sum_{m_2=0}^{k_2} P(k_1,m_1,k_2,m_2)=1.$$

The starting point is to consider the first two of the local balance equations for $P(\underline{N}^v,\underline{N}^s)$ given in Appendix A, namely

$$\frac{\alpha}{\alpha+\beta}\rho_p^v P(\underline{N}_p^{v-},\underline{N}_p^{s-}) = n_p^s P(\underline{N}^v,\underline{N}^s), \quad n_p^v,\ n_p^s \geq 1)$$

and

$$\frac{\beta}{\alpha+\beta}\rho_p^v P(\underline{N}_p^{v-},\underline{N}_p^{s}) = (n_p^v - n_p^s) P(\underline{N}^v,\underline{N}^s), \quad n_p^v \geq 1$$

where we have dropped the subscript $\mathcal{P}$ in $\underline{N}_{\mathcal{P}p}^{v-}$, $\underline{N}_{\mathcal{P}p}^{s-}$, and in the other vectors in order to simplify the notation; as usual, $\rho_p^v = F_p/\mu_p^v$. After summing both members of these equations with respect to all $(\underline{N}^v,\underline{N}^s)$ in the set $\Omega(k_1,m_1,k_2,m_2)$ defined above we obtain

$$\frac{\alpha}{\alpha+\beta}\rho_p^v \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(k_1,m_1,k_2,m_2)\cap\{n_p^v,n_p^s\geq 1\}} P(\underline{N}^{v-},\underline{N}^{s-}) = \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(k_1,m_1,k_2,m_2)} n_p^s P(\underline{N}^v,\underline{N}^s) \tag{D-1}$$

and

$$\frac{\beta}{\alpha+\beta}\rho_p^v \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(k_1,m_1,k_2,m_2)\cap\{n_p^v,n_p^s\geq 1\}} P(\underline{N}^{v-},\underline{N}^{s}) = \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(k_1,m_1,k_2,m_2)} (n_p^v - n_p^s) P(\underline{N}^v,\underline{N}^s) \tag{D-2}$$

where

$$\begin{aligned}
&\Omega(k_1,m_1,k_2,m_2)\cap\{n_p^v,n_p^s\geq 1\} \\
&= \Big\{(\underline{N}^v,\underline{N}^s)\ :\ 1\leq n_p^s \leq n_p^v,\ 0\leq n_q^s\leq n_q^v;\ q\neq p,\ q\in\mathcal{P}; \\
&\qquad \sum_{q\in\mathcal{P}_i, q\neq p} r_q^{(i)} n_q^s + r_p^{(i)}(n_p^s - I(p\in\mathcal{P}_i)) = m_i - r_p^{(i)} I(p\in\mathcal{P}_i) \\
&\qquad \sum_{q\in\mathcal{P}_i, q\neq p} r_q^{(i)} n_q^v + r_p^{(i)}(n_p^v - I(p\in\mathcal{P}_i)) = k_i - r_p^{(i)} I(p\in\mathcal{P}_i),\quad i=1,2\Big\}
\end{aligned}$$

The LHS of (D-2) becomes

$$\frac{\alpha}{\alpha+\beta}\rho_p^v\, P(k_1 - r_p^{(1)} I(p \in \mathcal{P}_1), m_1 - r_p^{(1)} I(p \in \mathcal{P}_1), k_2 - r_p^{(2)} I(p \in \mathcal{P}_2), m_2 - r_p^{(2)} I(p \in \mathcal{P}_2))$$

Note that if $m_i = 0$ then $p \notin \mathcal{P}_i$, (since $m_i = 0$ implies that $n_p^s = 0$, $\forall p \in \mathcal{P}_i$). The RHS of (D-2) can be written as

$$P(k_1, m_1, k_2, m_2) \cdot \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(k_1, m_1, k_2, m_2)} n_p^s \frac{P(\underline{N}^v, \underline{N}^s)}{P(k_1, m_1, k_2, m_2)}$$

where

$$P\left((\underline{N}^v, \underline{N}^s) \mid \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^v = k_i, \sum_{p \in \mathcal{P}_i} r_p^{(i)} n_p^s = m_i, i = 1,2\right)$$

$$= \begin{cases} \frac{P(\underline{N}^v, \underline{N}^s)}{P(k_1, m_1, k_2, m_2)}; & (\underline{N}^v, \underline{N}^s) \in \Omega(k_1, m_1, k_2, m_2) \\ 0; & \text{otherwise} \end{cases}$$

Since the RHS has thus taken effectively the form of the conditional expectation of n_p^s given (k_1, m_1, k_2, m_2), by combining the RHS and the LHS we conclude that

$$\frac{\alpha}{\alpha+\beta}\rho_p^v\, P\left(k_1 - r_p^{(1)} I(p \in \mathcal{P}_1), m_1 - r_p^{(1)} I(p \in \mathcal{P}_1), k_2 - r_p^{(2)} I(p \in \mathcal{P}_2), m_2 - r_p^{(2)} I(p \in \mathcal{P}_2)\right) =$$

$$= E[n_p^s | (k_1, m_1, k_2, m_2)] \cdot P(k_1, m_1, k_2, m_2) \qquad (D-3)$$

From equation (D-2) we can obtain in a similar way the equation

$$\frac{\beta}{\alpha+\beta}\rho_p^v P\left(k_1 - r_p^{(1)} I(p \in \mathcal{P}_1), m_1, k_2 - r_p^{(2)} I(p \in \mathcal{P}_2), m_2\right) =$$

$$= E[n_p^v - n_p^s | (k_1, m_1, k_2, m_2)] \cdot P(k_1, m_1, k_2, m_2) \qquad (D-4)$$

If we multiply equations (D-3) and (D-4) by $r_p^{(i)}$ and sum with respect to $p \in \mathcal{P}_i$, for $i = 1, 2$, we obtain

$P(k_1, m_1, k_2, m_2) =$

$$
=\begin{cases}
\frac{1}{m_1}\frac{\alpha}{\alpha+\beta}\Big[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c} r_p^{(1)}\rho_p^v P(k_1-r_p^{(1)}, m_1-r_p^{(1)}; k_2, m_2) + \sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2} r_p^{(1)}\rho_p^v P(k_1-r_p^{(1)}, m_1-r_p^{(1)}; k_2-r_p^{(2)}, m_2-r_p^{(2)})\Big], \\
\qquad\qquad\qquad \text{if } r_p^{(1)} \le m_1 \le k_1 \le Z_1, r_p^{(2)} \le m_2 \le k_2 \le Z_2; \\
\frac{1}{m_1}\cdot\frac{\alpha}{\alpha+\beta}\Big[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c} r_p^{(1)}\rho_p^v P(k_1 - r_p^{(1)}, m_1 - r_p^{(1)}; k_2, m_2)\Big], \\
\qquad\qquad\qquad \text{if } r_p^{(1)} \le m_1 \le k_1 \le Z_1, 0 = m_2 \le k_2 \le Z_2; \\
\frac{1}{m_2}\cdot\frac{\alpha}{\alpha+\beta}\Big[\sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2^c} r_p^{(2)}\rho_p^v P(k_1, m_1; k_2 - r_p^{(2)}, m_2 - r_p^{(2)})\Big], \\
\qquad\qquad\qquad \text{if } 0 = m_1 \le k_1 \le Z_1; r_p^{(2)} \le m_2 \le k_2 \le Z_2; \qquad (D-5) \\
\frac{1}{k_1-m_1}\cdot\frac{\beta}{\alpha+\beta}\Big[\sum_{p\in\mathcal{P}_2\cap\mathcal{P}_1^c} r_p^{(1)}\rho_p^v P(k_1-r_p^{(1)}, m_1; k_2, m_2) + \sum_{p\in\mathcal{P}_1\cap\mathcal{P}_2} r_p^{(1)}\rho_p^v P(k_1-r_p^{(1)}, m_1; k_2-r_p^{(2)}, m_2)\Big] \\
\qquad\qquad\qquad \text{if } 0 = m_1 < r_p^{(1)} \le k_1 \le Z_1, 0 = m_2 < r_p^{(2)} \le k_2 \le Z_2; \\
1; \qquad\qquad\qquad \text{if } k_1 = 0, m_1 = 0, k_2 = 0, m_2 = 0 \\
0; \qquad\qquad\qquad \text{if } k_1, m_1 \text{ are not positive integer multiples of } r_p^{(1)} \\
\qquad\qquad\qquad \text{or } k_2, m_2 \text{ are not positive integer multiples of } r_p^{(2)}
\end{cases}
$$

which provide the desirable recursions for evaluating $P(k_1, m_1, k_2, m_2)$ for all (k_1, m_1, k_2, m_2) after performing the normalization

$$P'(k_1, m_1, k_2, m_2) = P(k_1, m_1, k_2, m_2)/P(0, 0, 0, 0).$$

APPENDIX E

Proof of Proposition 6.1

(Pascal Approximation)

When $c = \infty$, we may rewrite equations (6.7) as

$$\hat{P}(k,m) = \begin{cases} \frac{\lambda_{1,k-1}}{k} \cdot \hat{P}(k-1,0), & m=0;\ 1 \le k \le \infty \\ \frac{\lambda_{2,m-1}}{m} \cdot \hat{P}(k-1,m-1), & 1 \le m \le k;\ 1 \le k \le \infty \end{cases}$$

where

$$\sum_{k=0}^{\infty} \sum_{m=0}^{k} \hat{P}(k,m) = 1.$$

To express $\hat{P}(k,m)$ as a function of $\hat{P}(0,0)$, we write

$$\hat{P}(k,m) = \begin{cases} \frac{\prod_{i=0}^{k-1} \lambda_{1,i}}{m!} \cdot \hat{P}(0,0), & m=0;\ 1 \le k \le \infty \\ \frac{\prod_{i=0}^{m-1} \lambda_{2,i}}{m!} \frac{\prod_{i=0}^{k-m-1} \lambda_{1,i}}{(k-m)!} \cdot \hat{P}(0,0), & 1 \le m \le k-1,\ 2 \le k \le \infty \\ \frac{\prod_{i=0}^{m-1} \lambda_{2,i}}{m!} \cdot \hat{P}(0,0), & m=k,\ 1 \le k \le \infty \end{cases}$$

and equation

$$\sum_{k=0}^{\infty} \sum_{m=0}^{k} \hat{P}(k,m) = 1$$

becomes

$$\hat{P}(0,0) + \sum_{k=1}^{\infty} \hat{P}(k,0) + \sum_{k=1}^{\infty} \hat{P}(k,k) + \sum_{k=2}^{\infty} \sum_{m=1}^{k-1} \hat{P}(k,m) = 1$$

which results in

$$\left[1 + \sum_{k=1}^{\infty} \frac{\prod_{i=0}^{k-1} \lambda_{1,i}}{k!} + \sum_{k=1}^{\infty} \frac{\prod_{i=0}^{k-1} \lambda_{2,i}}{k!} + \sum_{k=2}^{\infty} \sum_{m=1}^{k-1} \frac{\prod_{i=0}^{m-1} \lambda_{2,i}}{m!} \cdot \frac{\prod_{i=0}^{k-m-1} \lambda_{1,i}}{(k-m)!}\right] \cdot \hat{P}(0,0) = 1.$$

Let

$$a_j = \frac{\epsilon_j^2}{\sigma_j^2}$$

$$b_j = 1 - \frac{\epsilon_j}{\sigma_j^2}$$

for $j = 1, 2$. Then by definition,

$$\lambda_{j,i} = a_j + i \cdot b_j$$

for $j = 1, 2$. After certain manipulations, we have

$$\begin{aligned}
1 + \sum_{k=1}^{\infty} \left(\frac{\prod_{i=0}^{k-1} \lambda_{1,i}}{k!} + \frac{\prod_{i=0}^{k-1} \lambda_{2,i}}{k!} \right) + \sum_{k=2}^{\infty} \sum_{m=1}^{k-1} \frac{\prod_{i=0}^{m-1} \lambda_{2,i}}{m!} \frac{\prod_{i=0}^{k-m-1} \lambda_{1,i}}{(k-m)!} \\
= \left(1 + \sum_{k=1}^{\infty} \frac{\prod_{i=0}^{k-1} \lambda_{1,i}}{k!} \right) \cdot \left(1 + \sum_{k=1}^{\infty} \frac{\prod_{i=0}^{k-1} \lambda_{2,i}}{k!} \right) \\
= (1 - b_1)^{-a_1/b_1} \cdot (1 - b_2)^{-a_2/b_2}
\end{aligned}$$

Then

$$\hat{P}(0,0) = (1 - b_1)^{a_1/b_1} \cdot (1 - b_2)^{a_2/b_2}$$

Consider the moment generating function

$$\begin{aligned}
E[T^k S^m] &= \hat{P}(0,0) + \sum_{k=1}^{\infty} [\hat{P}(k,0) T^k + \hat{P}(k,k) T^k S^k] + \sum_{k=2}^{\infty} \sum_{m=1}^{k-1} \hat{P}(k,m) T^k S^m \\
&= \hat{P}(0,0) \cdot \left[1 + \sum_{k=1}^{\infty} \left(\frac{\prod_{i=0}^{k-1} (\lambda_{1,i} \cdot T)}{k!} + \frac{\prod_{i=0}^{k-1} (\lambda_{2,i} \cdot T \cdot S)}{k!} \right) \right. \\
&\quad \left. + \sum_{k=2}^{\infty} \sum_{m=1}^{k-1} \frac{\prod_{i=0}^{m-1} (\lambda_{2,i} \cdot T \cdot S)}{m!} \cdot \frac{\prod_{i=0}^{k-m-1} (\lambda_{1,i} \cdot T)}{(k-m)!} \right].
\end{aligned}$$

Note that

$$\lambda_{1,i} \cdot T = a_1 \cdot T + i b_1 \cdot T$$

$$\lambda_{2,i} \cdot TS = a_2 \cdot TS + i b_2 \cdot T \cdot S.$$

Therefore,

$$E[T^k S^m] = \left(\frac{1 - b_1 T}{1 - b_1} \right)^{-\frac{a_1}{b_1}} \left(\frac{1 - b_2 TS}{1 - b_2} \right)^{-\frac{a_2}{b_2}}.$$

Then the first and second moments of k and m are

$$E[k-m] = E[k] - E[m] = \left.\frac{\partial E[T^k S^m]}{\partial T}\right|_{T=S=1} - \left.\frac{\partial E[T^k S^m]}{\partial S}\right|_{T=S=1}$$

$$= \left(\frac{a_2}{1-b_2} + \frac{a_1}{1-b_1}\right) - \frac{a_2}{1-b_2}$$

$$= \frac{a_1}{1-b_1} = \epsilon_1.$$

$$E[m] = \left.\frac{\partial E[T^k S^m]}{\partial S}\right|_{T=S=1}$$

$$= \frac{a_2}{1-b_2} = \epsilon_2.$$

$$var(m) = E[m^2] - E[m]^2 = \left.\frac{\partial^2 E[T^k S^m]}{\partial S^2}\right|_{T=S=1} - E[m]^2$$

$$= \left(\frac{a_2^2 + a_2 b_2}{(1-b_2)^2} + \frac{a_2}{(1-b_2)}\right) - \left(\frac{a_2^2}{(1-b_2)}\right)^2$$

$$= \frac{a_2}{(1-b_2)^2} = \sigma_2^2.$$

$$var(k-m) = \left.\frac{\partial^2 E[T^k S^m]}{\partial T^2}\right|_{T=S=1} - 2\left.\frac{\partial^2 E[T^k S^m]}{\partial T \partial S}\right|_{T=S=1} + var(m) + E[k] \cdot (2E[m] - E[k])$$

$$= \frac{a_1}{(1-b_1)^2} = \sigma_1^2.$$

$$cov(k-m, m) = E[(k-m)m] - E[k-m] \cdot E[m]$$

$$= \left.\frac{\partial^2 E[T^k S^m]}{\partial T \partial S}\right|_{T=S=1} - E[k]E[m] - var(m)$$

$$= 0.$$

APPENDIX F

Proof of Proposition 6.2

(Pascal Approximation)

For the single-link, multi-rate, infinite capacity scheme, the steady-state probability of voice $P(\underline{N}^v, \underline{N}^s)$ in Section 3 is

$$P(\underline{N}^v, \underline{N}^s) = \frac{1}{G} \cdot \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}},$$

where

$$G = \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v, p \in \mathcal{P}}} \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}}$$

and it is approximated by

$$G \approx \prod_{p \in \mathcal{P}} \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}}.$$

which means that the voice states of the different paths are assumed mutually independent in this single-link analysis. This is motivated by the same interlink blocking independence assumption discussed in Sections 4.1 and 5.1. Therefore, we may define

$$\tilde{P}(\underline{N}^v, \underline{N}^s) = \prod_{p \in \mathcal{P}} p(n_p^v, n_p^s)$$

where

$$p(n_p^v, n_p^s) = \frac{(\rho_p^v)^{n_p^v} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v}}}{n_p^s!(n_p^v - n_p^s)!} \cdot \frac{1}{G_p}$$

and

$$G_p = \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} \frac{(\rho_p^v)^{n_p^v} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v}}}{n_p^s!(n_p^v - n_p^s)!}.$$

In the following we use $\tilde{P}(\underline{N}^v, \underline{N}^s)$ instead of $P(\underline{N}^v, \underline{N}^s)$ and $\tilde{P}(\underline{N}^v)$ instead of $P(\underline{N}^v)$ to distinguish the approximations from the exact values. Next we evaluate the first and second moments of $(\underline{N}^v, \underline{N}^s)$ as follows

$$E[n_v] = \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v, p \in \mathcal{P}}} \sum_{p \in \mathcal{P}} r_p n_p^v \cdot \tilde{P}(\underline{N}^v, \underline{N}^s) = \sum_{\substack{0 \le n_p^v \le \infty \\ p \in \mathcal{P}}} \sum_{p \in \mathcal{P}} r_p n_p^v \cdot \tilde{P}(\underline{N}^v)$$

where

$$\tilde{P}(\underline{N}^v) = \left[\prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^v!} \right] \Bigg/ \left[\sum_{\substack{0 \le n_p^v \le \infty \\ p \in \mathcal{P}}} \prod_{p \in \mathcal{P}} \frac{(\rho_p^v)^{n_p^v}}{n_p^v!} \right] = \prod_{p \in \mathcal{P}} p(n_p^v)$$

and

$$p(n_p^v) = \left[\frac{(\rho_p^v) n_p^v}{n_p^v!} \right] \Bigg/ \left[\sum_{0 \le n_p^v \le \infty} \frac{(\rho_p^v) n_p^v}{n_p^v!} \right].$$

$\tilde{P}(\underline{N}^v)$ is the approximate steady-state probability of state $\underline{N}^v$, where the state vector $\underline{N}^v$ represents the number of channels occupied by voice taking path p, for $p \in \mathcal{P}$. Combining these results yields

$$E[n_v] = \sum_{\substack{0 \le n_p^v \le \infty \\ p \in \mathcal{P}}} \sum_{p \in \mathcal{P}} r_p n_p^v \prod_{p \in \mathcal{P}} p(n_p^v) = \sum_{p \in \mathcal{P}} r_p \sum_{0 \le n_p^v \le \infty} n_p^v \, p(n_p^v) = \sum_{p \in \mathcal{P}} r_p \rho_p^v$$

and similarly

$$E[n_s] = \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v, p \in \mathcal{P}}} \sum_{p \in \mathcal{P}} r_p n_p^s \prod_{p \in \mathcal{P}} p(n_p^v, n_p^s) = \sum_{p \in \mathcal{P}} r_p \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} n_p^s \, p(n_p^v, n_p^s).$$

Note that using the definition of $p(n_p^v, n_p^s)$ above we can express $n_p^s \cdot p(n_p^v, n_p^s)$ as

$$n_p^s \cdot p(n_p^v, n_p^s) = \frac{(\rho_p^v)^{n_p^v - 1} \frac{\alpha^{n_p^s - 1} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v - 1}}}{(n_p^s - 1)!(n_p^v - n_p^s)!} \cdot \frac{\alpha \rho_p^v}{\alpha + \beta} \cdot \frac{1}{G_p}$$

and thus through summation obtain

$$\sum_{\substack{0\le n_p^v\le\infty \\ 0\le n_p^s\le n_p^v}} n_p^s p(n_p^v, n_p^s) = \sum_{\substack{1\le n_p^v\le\infty \\ 1\le n_p^s\le n_p^v}} n_p^s p(n_p^v, n_p^s) = \frac{\alpha}{\alpha+\beta}\rho_p^v.$$

Therefore,

$$E[n_s] = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}} r_p\rho_p^v.$$

Since n_v depends on $\underline{N}^v$ but not on $\underline{N}^s$, we can use directly the result of [6] for voice-only traffic (without silent periods):

$$var(n_v) = \sum_{p\in\mathcal{P}} r_p(\rho_p^v)^2.$$

Moreover,

$$E[n_s^2] = \sum_{\substack{0\le n_p^v\le\infty \\ 0\le n_p^s\le n_p^v, p\in\mathcal{P}}} \left(\sum_{p\in\mathcal{P}} r_p n_p^s\right)^2 \tilde{P}(\underline{N}^v, \underline{N}^s)$$

$$= \sum_{\substack{0\le n_p^v\le\infty \\ 0\le n_p^s\le n_p^v, p\in\mathcal{P}}} \left(\sum_{p\in\mathcal{P}} r_p^2 n_p^{s2} \sum_{p\in\mathcal{P}}\sum_{\substack{q\in\mathcal{P} \\ q\neq p}} r_p n_p^s r_q n_q^s\right) \tilde{P}(\underline{N}^v, \underline{N}^s)$$

$$= \sum_{p\in\mathcal{P}} r_p \sum_{\substack{0\le n_p^v\le\infty \\ 0\le n_p^s\le n_p^v}} n_p^{s2}\; p(n_p^v, n_p^s) + \sum_{p\in\mathcal{P}}\sum_{\substack{q\in\mathcal{P} \\ q\neq p}} r_p r_q \sum_{\substack{0\le n_q^v\le\infty \\ 0\le n_q^s\le n_q^v}} \sum_{\substack{0\le n_p^v\le\infty \\ 0\le n_p^s\le n_p^v}} n_p^v n_p^s \cdot p(n_p^v, n_p^s)\cdot p(n_q^v, n_q^s).$$

Then, since

$$\sum_{\substack{0\le n_p^v\le\infty\\0\le n_p^s\le n_p^v}} n_p^{s2} p(n_p^v, n_p^s)$$

$$= \sum_{\substack{1\le n_p^v\le\infty\\1\le n_p^s\le n_p^v}} (n_p^s - 1 + 1)\frac{\rho_p^{n_p^v-1}\frac{\alpha^{n_p^s-1}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-1}}}{(n_p^s-1)!(n_p^v-n_p^s)!}\cdot\frac{\alpha\rho_p^v}{\alpha+\beta}\cdot\frac{1}{G_p}$$

$$= \frac{\alpha}{\alpha+\beta}\rho_p^v\left[\sum_{\substack{1\le n_p^v\le\infty\\1\le n_p^s\le n_p^v}} (n_p^s-1)\frac{\rho_p^{n_p^v-1}\frac{\alpha^{n_p^s-1}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-1}}}{(n_p^s-1)!(n_p^v-n_p^s)!} + \sum_{\substack{1\le n_p^v\le\infty\\1\le n_p^s\le n_p^v}} \frac{(\rho_p^v)^{n_p^v-1}\frac{\alpha^{n_p^s-1}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-1}}}{(n_p^s-1)!(n_p^v-n_p^s)!}\right]\cdot\frac{1}{G_p}$$

$$= \frac{\alpha}{\alpha+\beta}\rho_p^v\left[\sum_{\substack{2\le n_p^v\le\infty\\2\le n_p^s\le n_p^v}} \frac{\rho_p^{n_p^v-2}\frac{\alpha^{n_p^s-2}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-2}}}{(n_p^s-2)!(n_p^v-n_p^s)!}\cdot\frac{\alpha}{\alpha+\beta}\rho_p^v + G_p\right]\cdot\frac{1}{G_p}$$

$$= \frac{\alpha}{\alpha+\beta}\rho_p^v\cdot\left(1+\frac{\alpha}{\alpha+\beta}\rho_p^v\right)$$

and

$$\sum_{\substack{0\le n_q^v\le\infty\\0\le n_q^s\le n_q^v}}\sum_{\substack{0\le n_p^v\le\infty\\0\le n_p^s\le n_p^v}} n_p^s n_q^s\cdot p(n_p^v,n_p^s)\cdot p(n_q^v,n_q^s) = \sum_{\substack{0\le n_p^v\le\infty\\0\le n_p^s\le n_p^v}} n_p^s\cdot p(n_p^v,n_p^s)\sum_{\substack{0\le n_q^v\le\infty\\0\le n_q^s\le n_q^v}} n_q^s\cdot p(n_q^v,n_q^s)$$

$$= \frac{\alpha}{\alpha+\beta}\rho_p^v\cdot\frac{\alpha}{\alpha+\beta}\rho_q^v$$

we obtain

$$E[n_s^2] = \sum_{p\in\mathcal{P}} r_p\left(\frac{\alpha}{\alpha+\beta}\rho_p^v + \left(\frac{\alpha}{\alpha+\beta}\rho_p^v\right)^2\right) + \sum_{p\in\mathcal{P}}\sum_{\substack{q\in\mathcal{P}\\q\ne p}} r_p r_q \rho_p^v\rho_q^v\left(\frac{\alpha}{\alpha+\beta}\right)^2$$

$$= \sum_{p\in\mathcal{P}} r_p\frac{\alpha}{\alpha+\beta}\rho_p^v + \left(\sum_{p\in\mathcal{P}} r_p\frac{\alpha}{\alpha+\beta}\rho_p^v\right)^2.$$

From the above expressions for $E[n_s]$ and $E[n_s^2]$ we can easily derive that

$$var(n_s) = E[n_s^2] - (E[n_s])^2 = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}} r_p^2 \rho_p^v.$$

Similarly, we can write

$$E[(n_v - n_s)^2] = \sum_{\substack{0\le n_p^v \le \infty \\ 0\le n_p^s \le n_p^v}} \left(\sum_{p\in\mathcal{P}} r_p(n_p^v - n_p^s)\right)^2 \tilde{P}(\underline{N}^v, \underline{N}^s)$$

$$= \sum_{p\in\mathcal{P}} r_p^2 \sum_{\substack{0\le n_q^v \le \infty \\ 0\le n_q^s \le n_q^v}} (n_p^v - n_p^s)^2 p(n_p^v, n_p^s)$$

$$+ \sum_{p\in\mathcal{P}} \sum_{\substack{q\in\mathcal{P} \\ q\neq p}} r_p r_q \cdot \sum_{\substack{0\le n_p^v \le \infty \\ 0\le n_p^s \le n_p^v}} (n_p^v - n_p^s) p(n_p^v, n_p^s) \sum_{\substack{0\le n_q^v \le \infty \\ 0\le n_q^s \le n_q^v}} (n_q^v - n_q^s) p(n_p^v, n_p^s)$$

and since

$$\sum_{\substack{0\le n_p^v \le \infty \\ 0\le n_p^s \le n_p^v}} (n_p^v - n_p^s)^2 (n_p^v, n_p^s) = \sum_{\substack{1\le n_p^v \le \infty \\ 0\le n_p^s \le n_p^v - 1}} (n_p^v - n_p^s - 1 + 1) \frac{(\rho_p^v - 1)\frac{\alpha^{n_p^s}\beta^{n_p^v - n_p^s - 1}}{(\alpha+\beta)^{n_p^v - 1}}}{(n_p^s)!(n_p^v - n_p^s - 1)!} \frac{\beta}{\alpha+\beta}\rho_p^v \cdot \frac{1}{G_p}$$

$$= \left[\sum_{\substack{2\le n_p^v \le \infty \\ 0\le n_p^s \le n_p^v - 2}} \frac{\rho_p^{n_p^v - 2}\frac{\alpha^{n_p^s}\beta^{n_p^v - n_p^s - 2}}{(\alpha+\beta)^{n_p^v - 2}}}{(n_p^s)!(n_p^v - n_p^s - 1)!} \frac{\beta}{\alpha+\beta}\rho_p^v + G_p \right] \cdot \frac{\beta}{\alpha+\beta}\rho_p^v \cdot \frac{1}{G_p}$$

$$= \frac{\beta}{\alpha+\beta}\rho_p^v \cdot \left(1 + \frac{\beta}{\alpha+\beta}\rho_p^v\right)$$

we derive that

$$E[(n_v - n_s)^2] = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}} r_p^2 \rho_p^v + \left(\frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}} r_p \rho_p^v\right)^2$$

and conclude that

$$var(n_v - n_s) = E[(n_v - n_s)^2] - (E[n_v - n_s])^2 = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}} r_p^2 \rho_p^v.$$

Finally, we write

$$E[n_v \cdot n_s] = \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} \left(\sum_{p \in \mathcal{P}} r_p n_p^v \right) \left(\sum_{p \in \mathcal{P}} r_p n_p^s \right) \tilde{P}(\underline{N}^v, \underline{N}^s)$$

$$= \sum_{p \in \mathcal{P}} r_p^2 \sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} n_p^v n_p^s p(n_p^v, n_p^s) \sum_{p \in \mathcal{P}} \sum_{\substack{q \in \mathcal{P} \\ q \ne p}} r_p r_q \left(\sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} n_p^s p(n_p^v, n_p^s) \right) \left(\sum_{\substack{0 \le n_q^v \le \infty \\ 0 \le n_q^s \le n_q^v}} n_q^v p(n_q^v, n_q^s) \right)$$

and since we have that

$$\sum_{\substack{0 \le n_p^v \le \infty \\ 0 \le n_p^s \le n_p^v}} n_p^v n_p^s p(n_p^v, n_p^s) = \frac{\alpha}{\alpha+\beta} \rho_p^v \cdot \left[\sum_{\substack{0 \le n_p^v \le \infty \\ 1 \le n_p^s \le n_p^v}} (n_p^v + 1) \frac{\rho_p^{n_p^v} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v}}}{(n_p^s)!(n_p^v - n_p^s)!} \right] \cdot \frac{1}{G_p}$$

$$= \frac{\alpha}{\alpha+\beta} \rho_p^v \left[\sum_{0 \le n_p^v \le \infty} n_p^v p(n_\rho^v) + 1 \right]$$

$$= \frac{\alpha}{\alpha+\beta} \rho_p^v (1 + \rho_p^v)$$

we derive that

$$E[n_v \cdot n_s] = \sum_{p \in \mathcal{P}} r_p^2 \frac{\alpha}{\alpha+\beta} (\rho_p^v + \rho_p^{v2}) \sum_{p \in \mathcal{P}} \sum_{\substack{q \in \mathcal{P} \\ q \ne p}} r_p r_q \frac{\alpha}{\alpha+\beta} \rho_p^v \rho_q^v$$

$$= \frac{\alpha}{\alpha+\beta} \sum_{p \in \mathcal{P}} r_p^2 \rho_p^v + \frac{\alpha}{\alpha+\beta} \left(\sum_{p \in \mathcal{P}} r_p \rho_p^v \right)^2$$

and conclude that

$$cov[n_v, n_s] = E[n_v \cdot n_s] - E[n_v] \cdot E[n_s] = \frac{\alpha}{\alpha+\beta} \sum_{p \in \mathcal{P}} r_p^2 \rho_p^v.$$

Consequently, since

$$cov(n_v - n_s, n_s) = E[(n_v - n_s)n_s] - E[n_v - n_s] \cdot E[n_s] = cov(n_v, n_s) - var(n_s)$$

and as we established above $cov(n_v, n_s) = var(n_s)$, we also obtain that

$$cov(n_v - n_s, n_s) = 0.$$

APPENDIX G

The Knapsack and Pascal Approximations to the Probabilities of Voice Blocking and Data Queueing Are Identical for Networks with Single-Rate Traffic

In this appendix we prove the validity of the claim made in the title. We show that the knapsack and Pascal methods provide identical approximations to the main performance measures of this report: the probability of blocking voice (Section 4) and the probability of queueing data (Sections 5 and 6). The same is true for the average queueing delay of data (Section 7), as well as for the probability distribution of the data queueing delay, but we omit the proof since it resembles the one for the probability of data queueing.

The single-rate case is characterized by

$$r_p = 1 \text{ for all } p \in \mathcal{P}$$

where 1 is the required data rate (bandwidth) of voice; the required bandwidth for data is then $1/r$ (where $r \geq 1$).

G.1 Probability of Voice Blocking for Single-Rate Traffic

Knapsack Approximation

In the single-rate case, the recursion of equations (4.6a)-(4.6b), which provides the weights necessary for the knapsack approximation to the voice blocking probability of (4.5), reduces to

$$w(n) = \frac{1}{n}\left(\sum_{p \in \mathcal{P}_l} \rho_p^v\right) w(n-1) \text{ for } n = 1, 2, \ldots, c_l.$$

Pascal Approximation

In the single-rate case, the recursion of equations (4.11b)-(4.12b), which provides the weights necessary for the Pascal approximation to the voice blocking probability of (4.13b), reduces to

$$q'(n) = \frac{\lambda_{n-1}}{n} q'(n-1) \text{ for } n = 1, 2, \ldots, c_l$$

where

$$\lambda_{n-1} = \frac{\epsilon^2}{\sigma^2} + (n-1)\left(1 - \frac{\epsilon}{\sigma^2}\right)$$

with

$$\epsilon = E\{n_v\} = \sum_{p \in \mathcal{P}_l} r_p \rho_p^v = \sum_{p \in \mathcal{P}_l} \rho_p^v$$

and

$$\sigma^2 = var\{n_v\} = \sum_{p \in \mathcal{P}_l} r_p^2 \rho_p^v = \sum_{p \in \mathcal{P}_l} \rho_p^v.$$

Since $\epsilon = \sigma^2$, λ_{n-1} simplifies to

$$\lambda_{n-1} = \epsilon = \sum_{p \in \mathcal{P}_l} \rho_p^v$$

resulting in

$$q'(n) = \frac{1}{n}\left(\sum_{p \in \mathcal{P}_l} \rho_p^v\right) q'(n-1).$$

Since as we know

$$w(0) = q'(0) = 1$$

comparison of the above simplified recursions for the knapsack and Pascal approximations shows that they are identical, that is

$$w(n) = q'(n) \ \text{ for } \ n = 0, 1, \ldots, c_l.$$

Therefore, since the form of the approximations in (4.5) and (4.13b) is the same except for the $w(n)$ and $q'(n)$, the two approximations coincide in the single-rate case.

G.2 Probability of Data Queueing for Single-Rate Traffic

Knapsack Approximation

In the single-rate case, the recursion providing the quantities $P'(k,m)$ of (5.10a) involved in the knapsack approximation to the probability of data queueing given by (5.9) reduces to

$$P'(k,m) = \begin{cases} \frac{1}{m}\left([\alpha/(\alpha+\beta)] \sum_{p \in \mathcal{P}_l} \rho_p^v\right) P'(k-1, m-1) & 1 \le m \le k \le c_l \\ \frac{1}{k}\left([\beta/(\alpha+\beta)] \sum_{p \in \mathcal{P}_l} \rho_p^v\right) P'(k-1, 0) & 0 = m < k \le c_l. \end{cases}$$

Pascal Approximation

Similarly, the recursion providing the corresponding quantities $P'(k,m)$ of (6.7b) involved in the Pascal approximation to the probability of data queueing given by (6.21) reduces in the single-rate case to

$$\hat{P}'(k,m) = \begin{cases} (\lambda_{2,m-1}/m)\hat{P}'(k-1,m-1) & 1 \le m \le k \le c_l \\ (\lambda_{1,k-1}/k)\hat{P}'(k-1,0) & m = 0 \end{cases}$$

where

$$\lambda_{2,m-1} = \frac{\epsilon_2^2}{\sigma_2^2} + (m-1)\left(1 - \frac{\epsilon_2}{\sigma_2^2}\right)$$

with

$$\epsilon_2 = E\{m\} = E\{n_s\} = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l}\rho_p^v$$

and

$$\sigma_2^2 = var\{m\} = var\{n_s\} = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p^2\rho_p^v = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l}\rho_p^v.$$

Since $\epsilon_2 = \sigma_2^2$, $\lambda_{2,m-1}$ simplifies to

$$\lambda_{2,m-1} = \epsilon_2 = \frac{\alpha}{\alpha+\beta}\sum_{p\in\mathcal{P}_l}\rho_p^v.$$

Similarly,

$$\lambda_{1,k-1} = \frac{\epsilon_1^2}{\sigma_1^2} + (k-1)\left(1 - \frac{\epsilon_1}{\sigma_1^2}\right)$$

and since

$$\epsilon_1 = E\{k-m\} = E\{n_v\} = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p\rho_p^v = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l}\rho_p^v$$

and

$$\sigma_1^2 = var\{k-m\} = var\{n_v\} = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l} r_p^2\rho_p^v = \frac{\beta}{\alpha+\beta}\sum_{p\in\mathcal{P}_l}\rho_p^v$$

imply that $\epsilon_1 = \sigma_1^2$, $\lambda_{1,k-1}$ simplifies to

$$\lambda_{1,k-1} = \epsilon_1 = \frac{\beta}{\alpha + \beta} \sum_{p \in \mathcal{P}_l} \rho_p^v.$$

Using the above simplified expressions for $\lambda_{1,k-1}$ and $\lambda_{2,m-1}$ in the recursion for $\hat{P}'(k,m)$ results in an expression whose functional form is identical to that obtained for $P'(k,m)$ in the knapsack recursion earlier in this subsection after the simplifications. Since we also have that

$$P'(0,0) = \hat{P}'(0,0) = 1$$

the knapsack and Pascal recursions are completely identical and thus

$$P'(k,m) = \hat{P}'(k,m) \text{ for all } 0 \le m \le k \le c_l.$$

Moreover, since the expressions providing the approximations to the probability of data queueing in (5.9) and (6.21) have the same functional form for both the knapsack and Pascal recursions, we deduce that the approximations in question are identical for the single-rate case.

APPENDIX H

Proof of Proposition 10.1

The starting point is the definition

$$W^d = \sum_{l \in \mathcal{L}} \gamma_l^d \rho_l^d \cdot \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega} P(\underline{N}^v, \underline{N}^s) \cdot \sum_{n_l^d = 0}^{c_l' - 1} P(n_l^d | c_l') \cdot I(c_l' > 0)$$

$$= \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega} P(\underline{N}^v, \underline{N}^s) \cdot w^d(\underline{\gamma}^d, \underline{\rho}^d, \underline{c})$$

from which we obtain

$$\frac{\partial W^d}{\partial \rho_p^v} = \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega} \frac{\partial P(\underline{N}^v, \underline{N}^s)}{\partial \rho_p^v} \cdot w^d(\underline{\gamma}^d, \underline{\rho}^d, \underline{c}).$$

Define

$$f_p = \frac{(\rho_p^v)^{n_p^v}}{n_p^s!(n_p^v - n_p^s)!} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha + \beta)^{n_p^v}}$$

and $G(\underline{c})$ as in (3.3); then from (3.2) we have

$$\frac{\partial P(\underline{N}^v, \underline{N}^s)}{\partial \rho_p^v} = \frac{\partial}{\partial \rho_p^v} \frac{1}{G(\underline{c})} \cdot \prod_{p \in \mathcal{P}} f_p + \frac{1}{G(\underline{c})} \cdot \prod_{q \in \mathcal{P} q \neq p} f_q \cdot \frac{\partial f_p}{\partial \rho_p^v}.$$

Moreover,

$$\frac{\partial f_p}{\partial \rho_p^v} = \frac{(\rho_p^v)^{n_p^v - 1} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v}}}{n_p^s!(n_p^v - n_p^s)!} \cdot (n_p^s + n_p^v - n_p^s)$$

$$= \frac{\alpha}{\alpha + \beta} \cdot \frac{(\rho_p^v)^{n_p^v - 1} \frac{\alpha^{n_p^s - 1} \beta^{n_p^v - n_p^s}}{(\alpha+\beta)^{n_p^v - 1}}}{(n_p^s - 1)!(n_p^v - n_p^s)!} \cdot I(n_p^s > 0)$$

$$+ \frac{\beta}{\alpha + \beta} \cdot \frac{\rho_p^{v\, n_p^v - 1} \frac{\alpha^{n_p^s} \beta^{n_p^v - n_p^s - 1}}{(\alpha+\beta)^{n_p^s - 1}}}{n_p^s!(n_p^v - n_p^s - 1)!} \cdot I(n_p^v - n_p^s > 0)$$

and

$$\frac{\partial}{\partial \rho_p^v} \frac{1}{G(\underline{c})} = -[G(\underline{c})]^{-2} \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c})} \prod_{q \in \mathcal{P} q \neq p} f_q \cdot \frac{\partial f_p}{\partial \rho_p^v}.$$

Therefore

$$\frac{\partial W^d}{\partial \rho_p^v} = -[G(\underline{c})]^{-2} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \prod_{p\in\mathcal{P}} f_p \cdot w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c})$$

$$\left\{ \frac{\alpha}{\alpha+\beta} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \left[\prod_{q\in\mathcal{P}q\neq p} f_q \frac{(\rho_p^v)^{n_p^v-1} \frac{\alpha^{n_p^s-1}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-1}}}{(n_p^s-1)!(n_p^v-n_p^s)!} I(n_p^s>0) \right] \right.$$

$$\left. + \frac{\beta}{\alpha+\beta} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \left[\prod_{q\in\mathcal{P}q\neq p} f_q \frac{(\rho_p^v)^{n_p^v-1} \frac{\alpha^{n_p^s}\beta^{n_p^v-n_p^s-1}}{(\alpha+\beta)^{n_p^v-1}}}{n_p^s!(n_p^v-n_p^s-1)!} \cdot I(n_p^v-n_p^s>0) \right] \right\}$$

$$+[G(\underline{c})]^{-1} \left\{ \frac{\alpha}{\alpha+\beta} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \left[\prod_{q\in\mathcal{P}q\neq p} f_q \frac{(\rho_p^v)^{n_p^v-1} \frac{\alpha^{n_p^s-1}\beta^{n_p^v-n_p^s}}{(\alpha+\beta)^{n_p^v-1}}}{(n_p^s-1)!(n_p^v-n_p^s)!} I(n_p^s>0) w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c}) \right] \right.$$

$$\left. + \frac{\beta}{\alpha+\beta} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \left[\prod_{q\in\mathcal{P}q\neq p} f_q \frac{(\rho_p^v)^{n_p^v-1} \frac{\alpha^{n_p^s}\beta^{n_p^v-n_p^s-1}}{(\alpha+\beta)^{n_p^v-1}}}{n_p^s!(n_p^v-n_p^s-1)!} I(n_p^v>n_p^s) w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c}) \right] \right\}$$

The above expression can be put in the simpler form

$$\frac{\partial W^d}{\partial \rho_p^v} = -[G(\underline{c})]^{-2} \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c})} \left[\frac{\alpha}{\alpha+\beta} \cdot (A) + \frac{\beta}{\alpha+\beta}(B) \right] \cdot \prod_{p\in\mathcal{P}} f_p \cdot w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c})$$

$$+ G(\underline{c})^{-1} \left[\frac{\alpha}{\alpha+\beta} \cdot (C) + \frac{\beta}{\alpha+\beta}(D) \right]$$

where the terms (A), (B), (C), (D) are appropriately defined.

The term (A) can be obtained as

$$(A) = \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c}-r_p\underline{e}_p)} \prod_{p\in\mathcal{P}} f_p = G(\underline{c}-r_p\underline{e}_p).$$

In order to simplify (B), let us consider $\Omega(\underline{c})\bigcap\{n_p^v > n_p^s\}$, which can be rewritten as

$$\Big\{(\underline{N}^v,\underline{N}^s)|0 \le \sum_{q\in\mathcal{P}_l} r_q n_q^v \le c_l, l\in\mathcal{L}; 0\le \sum_{q\in\mathcal{P}_l} r_q n_q^s \le \sum_{q\in\mathcal{P}_l} r_q n_q^v, l\notin p,$$

$$0\le \sum_{q\in\mathcal{P}_l} r_q n_q^s < \sum_{q\in\mathcal{P}_l} r_q n_q^v, l\in p, l\in\mathcal{L}\Big\} =$$

$$= \Big\{(\underline{N}^v,\underline{N}^s)|0 \le \sum_{q\in\mathcal{P}_l} r_q n_q^v \le c_l, 0\le \sum_{q\in\mathcal{P}_l} r_q n_q^s \le \sum_{q\in\mathcal{P}_l} r_q n_q^v, l\notin p;$$

$$0\le \sum_{q\in\mathcal{P}_l q\neq p} r_q n_q^v + r_p(n_p^v-1) \le c_l - r_p, 0\le \sum_{q\in\mathcal{P}_l} r_q n_q^s \le \sum_{q\in\mathcal{P}_l q\neq p} r_q n_q^v + r_p(n_p^v-1), l\in p\Big\}.$$

Therefore $\Omega(\underline{c})\bigcap\{n_p^v > n_p^s\}$ for vector $(\underline{N}^v,\underline{N}^s)$ is identical to $\Omega(\underline{c} - r_p\underline{e}_p)$ for vector $(\underline{N}_p^{v-},\underline{N}^s)$, where $\underline{N}_p^{v-} = \underline{N}^v - \underline{e}_p$, and (B) can be represented as

$$(B) = G(\underline{c} - r_p\underline{e}_p).$$

From a similar argument as for obtaining (B) and from the fact that

$$c_l' = c_l - \sum_{q\in\mathcal{P}_l} r_q n_q^v + \sum_{q\in\mathcal{P}_l} r_q n_q^s$$

$$= (c_l - r_p) - \left[\sum_{q\in\mathcal{P}_l q\neq p} r_q n_q^v + r_p(n_p^v - 1)\right] + \sum_{q\in\mathcal{P}_l} r_q n_q^s, \quad l\in p,$$

we can rewrite (D) as

$$(D) = G(\underline{c} - r_p\underline{e}_p)\cdot W^d(\underline{c} - r_p\underline{e}_p).$$

Also following similar arguments as in simplifying (A), we can obtain (C) as

$$(C) = \sum_{(\underline{N}^v,\underline{N}^s)\in\Omega(\underline{c}-r_p\underline{e}_p)} \prod_{p\in\mathcal{P}} f_p w^d(\underline{\gamma}^d,\underline{\rho}^d,\underline{c}).$$

Therefore

$$\frac{\partial W^d}{\partial \rho_p^v} = -[G(\underline{c})]^{-2} \cdot G(\underline{c} - r_p \underline{e}_p) \cdot \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c})} \prod_{p \in \mathcal{P}} f_p \cdot w^d(\underline{\gamma}^d, \underline{\rho}^d, \underline{c})$$

$$+ \frac{1}{G(\underline{c})} \left[\frac{\alpha}{\alpha+\beta} \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c} - r_p \underline{e}_p)} \prod_{p \in \mathcal{P}} f_p w^d(\underline{\gamma}^d, \underline{\rho}^d, \underline{c}) + \frac{\beta}{\alpha+\beta} G(\underline{c} - r_p \underline{e}_p) W^d(\underline{c} - r_p \underline{e}_p) \right]$$

$$= \frac{G(\underline{c} - r_p \underline{e}_p)}{G(\underline{c})} \left\{ \frac{\beta}{\alpha+\beta} W^d(\underline{c} - r_p \underline{e}_p) - W^d(\underline{c}) \right.$$

$$\left. + \frac{\alpha}{\alpha+\beta} \cdot \frac{1}{G(\underline{c} - r_p \underline{e}_p)} \cdot \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c} - r_p \underline{e}_p)} \prod_{p \in \mathcal{P}} f_p \cdot w^d(\underline{\gamma}^d, \underline{\rho}^d, \underline{c}) \right\}$$

$$= (1 - B_p) \left\{ \frac{\beta}{\alpha+\beta} W^d(\underline{c} - r_p \underline{e}_p) - W^d(\underline{c}) \right.$$

$$\left. + \frac{\alpha}{\alpha+\beta} \sum_{(\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c} - r_p \underline{e}_p)} P_{(\underline{c} - r_p \underline{e}_p)}(\underline{N}^v, \underline{N}^s) \cdot \sum_{l \in \mathcal{L}} \gamma_l^d \rho_l^d \bar{P}(\rho_l^d, c_l') \cdot I(c_l' > 0) \right\}$$

where

$$c_l' = c_l - \sum_{p \in \mathcal{P}_l} r_p n_p^v + \sum_{p \in \mathcal{P}_l} r_p n_p^s, \quad (\underline{N}^v, \underline{N}^s) \in \Omega(\underline{c} - r_p \underline{e}_p).$$

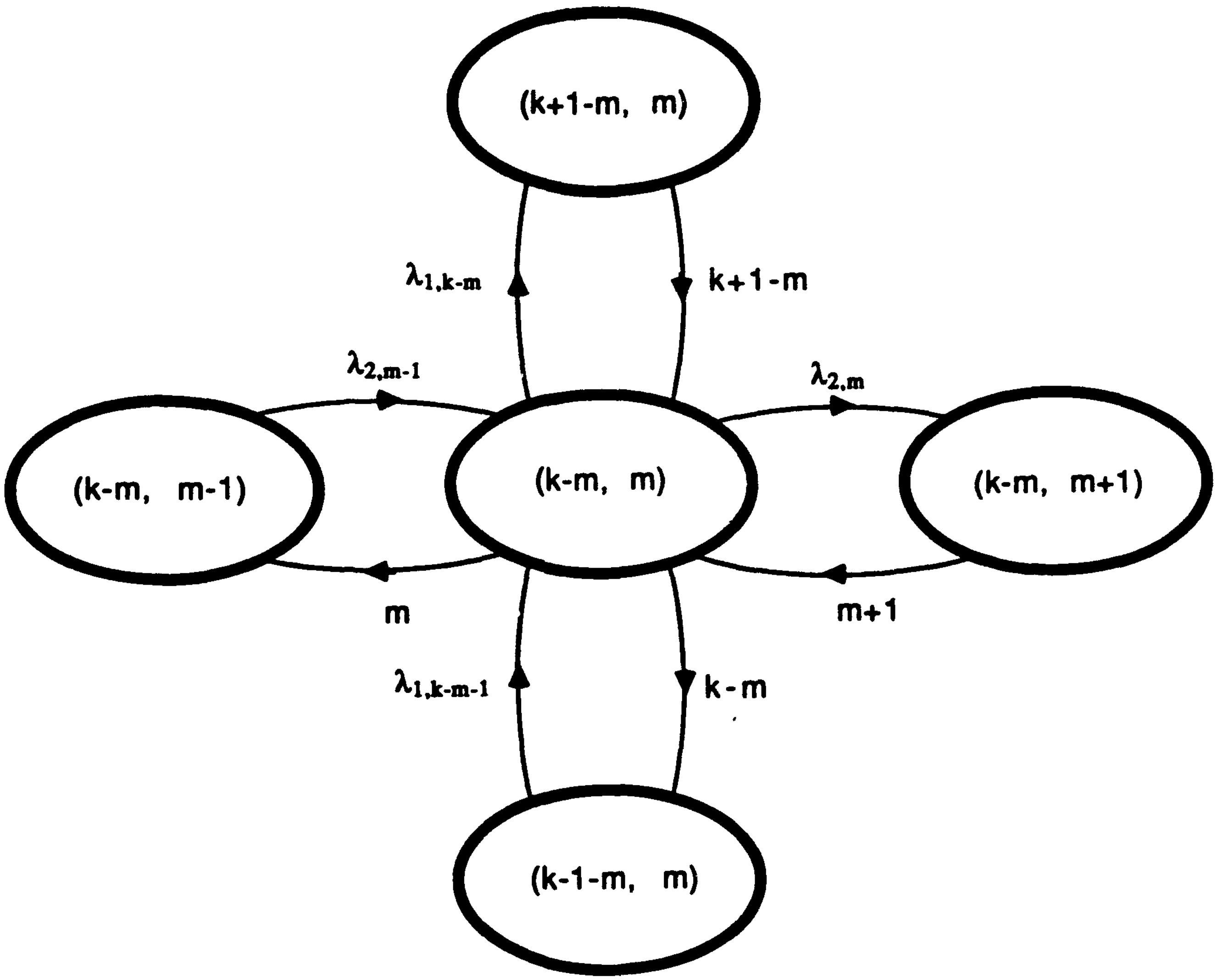

Figure 1. Two-Dimensional Birth-Death Process Used for the Pascal Approximation

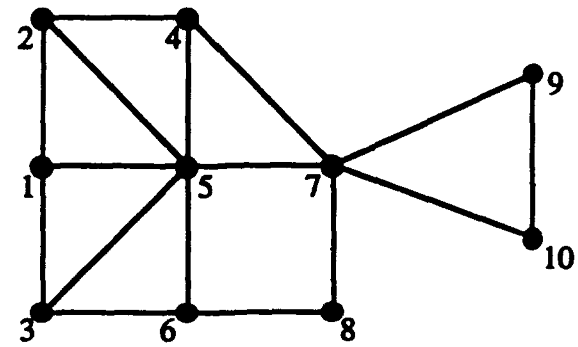

Figure 2a. The ten-node multihop radio network of [4]

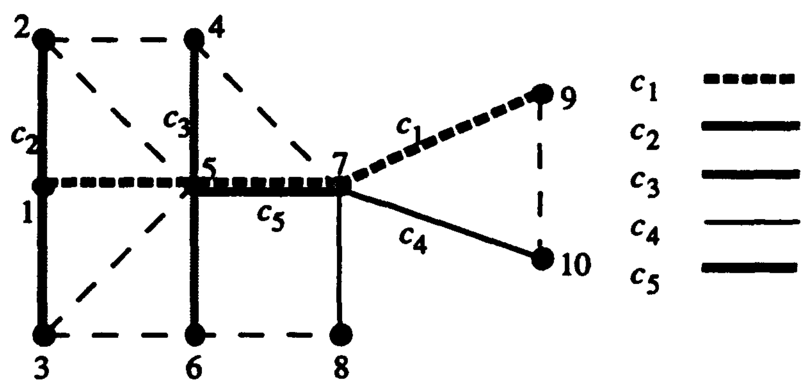

Figure 2b. The five superimposed circuits on the radio network of Figure 2a

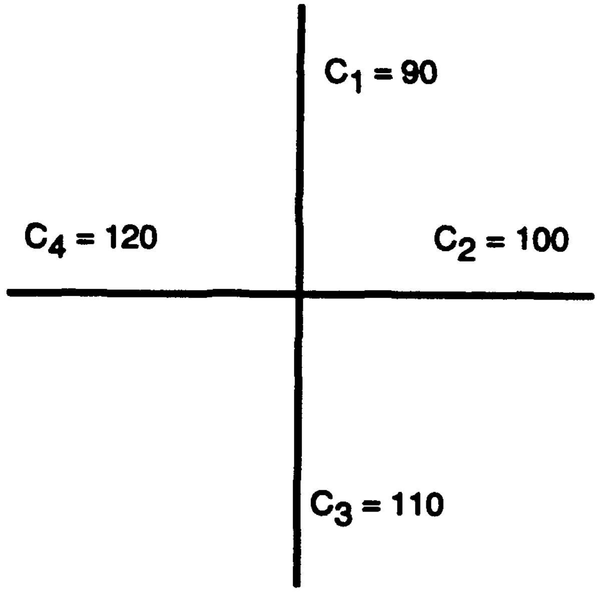

Figure 3a. The multi-rate star network of [3]

Figure 3b. The twelve voice circuits of the network of Figure 3a and their bandwidth requirements

Voice Circuit	Links Used	Bandwidth Required
1	1,2	1
2	1,3	1
3	1,4	1
4	2,3	1
5	2,4	1
6	3,4	1
7	1,2	5
8	1,3	5
9	1,4	5
10	2,3	5
11	2,4	5
12	3,4	5

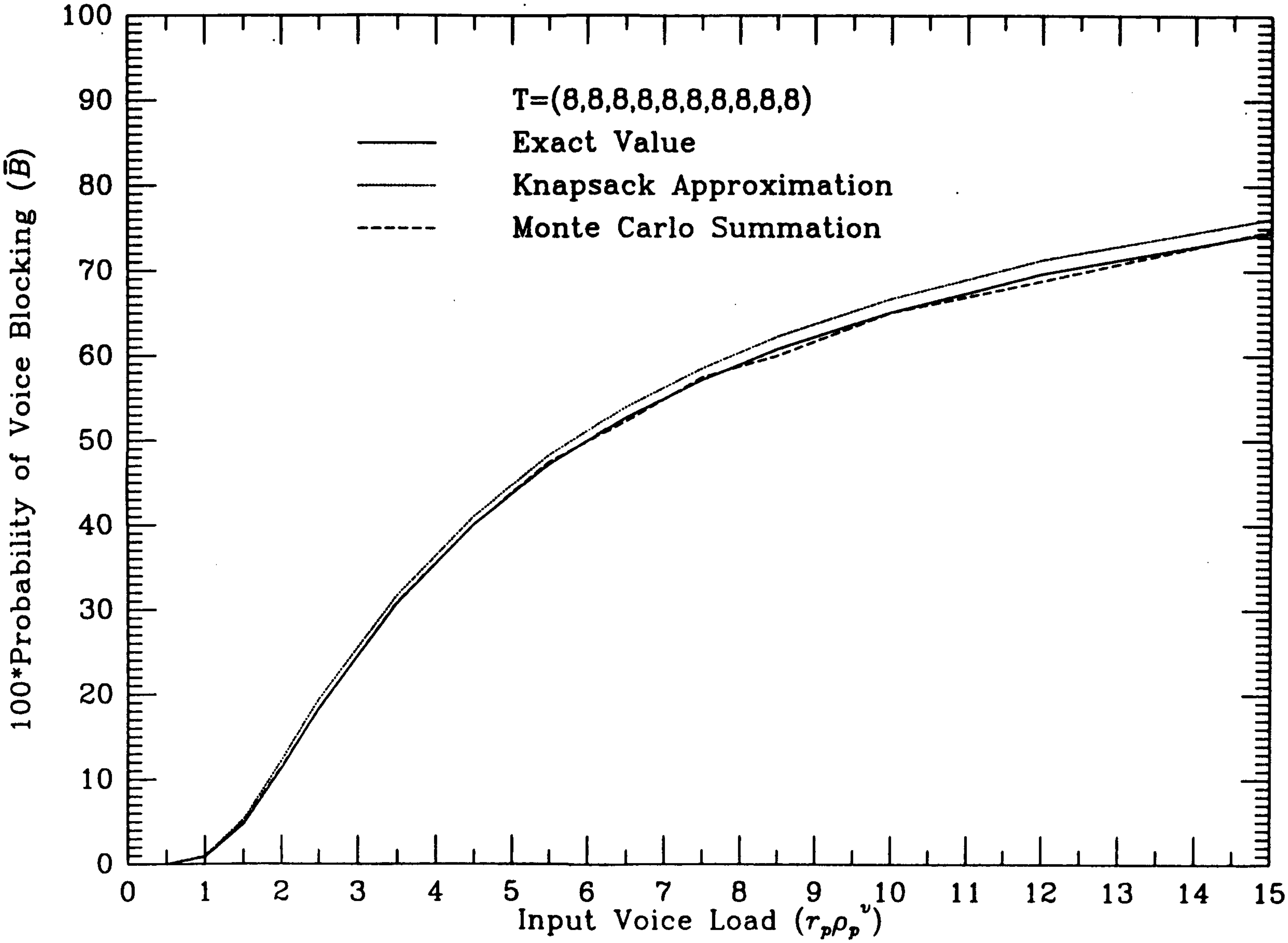

Figure 4. Exact value and approximations to the probability of voice blocking versus the offered voice load for the radio network model: no admission control

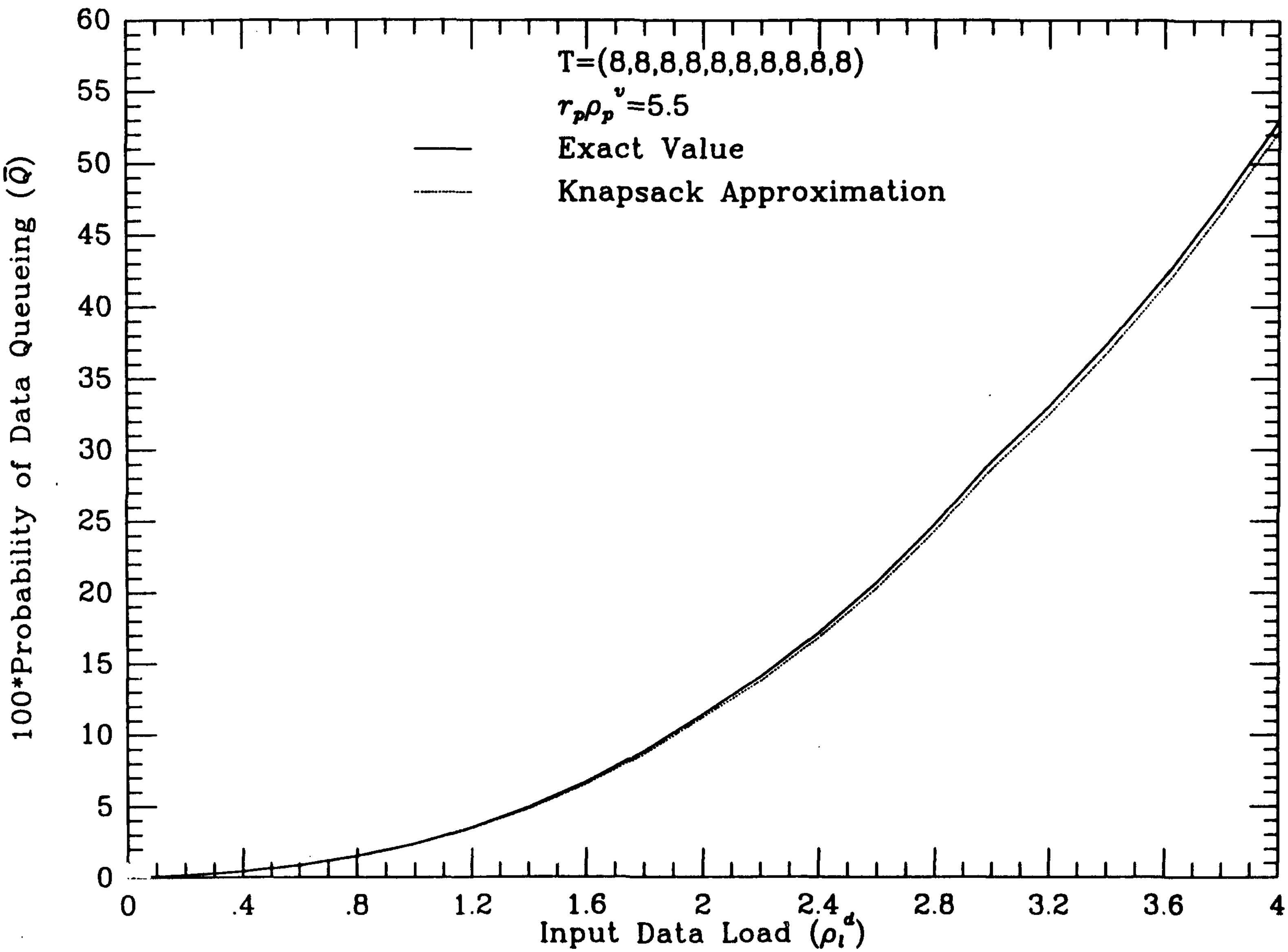

Figure 5. Exact value and approximations to the probability of data queueing versus the offered data load for the radio network model: no admission control

Table 1. Probability of Voice Blocking for Different Traffic Loads, Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$, Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$, and no Admission Control

Offered Voice Load $\bar{\rho}^v$	$\bar{B}$			
	Exact Value	Monte Carlo Summation	Knapsack Approximation	Error %
0.1	0.000000	(0.000000, 0.000000)	0.000000	–
0.5	0.000160	(0.000096, 0.000224)	0.000174	8.75
1.0	0.008654	(0.008355, 0.009262)	0.009802	13.26
1.5	0.047401	(0.044829, 0.061187)	0.053400	12.66
2.0	0.112365	(0.107544, 0.121025)	0.122152	8.71
2.5	0.183607	(0.179594, 0.185808)	0.194000	5.66
3.5	0.307931	(0.302761, 0.316174)	0.317236	3.02
4.5	0.401453	(0.385203, 0.417037)	0.411132	2.41
5.5	0.472130	(0.434905, 0.514719)	0.483226	2.35
6.5	0.527213	(0.505836, 0.540619)	0.539849	2.39
7.5	0.571417	(0.546001, 0.603585)	0.585356	2.44
8.5	0.607742	(0.534647, 0.646869)	0.622658	2.45
10.0	0.651634	(0.616248, 0.705557)	0.667461	2.43
15.0	0.745317	(0.676021, 0.778351)	0.761318	2.15

Table 2. Probability of Voice Blocking at Each Path for Different Traffic Loads, Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8,8)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice Path p	Offered Voice Load ρ_p^v	B_p			
		Exact Value	Monte Carlo Summation	Knapsack Approximation	Error %
1	1.0	0.014202	(0.011101, 0.018019)	0.016567	16.65
2	1.0	0.000730	(0.000000, 0.001098)	0.000837	14.66
3	1.0	0.007371	(0.005950, 0.011421)	0.007925	7.52
4	1.0	0.007371	(0.003915, 0.008254)	0.007925	7.52
5	1.0	0.013596	(0.010745, 0.017536)	0.015753	15.86
1	5.5	0.701485	(0.688131, 0.717825)	0.703886	0.34
2	5.5	0.209734	(0.174270, 0.210641)	0.247334	17.93
3	5.5	0.409218	(0.379417, 0.428384)	0.413480	1.04
4	5.5	0.409218	(0.384999, 0.433277)	0.413135	0.96
5	5.5	0.630995	(0.617493, 0.652789)	0.638292	1.16
1	10	0.885902	(0.858356, 0.887236)	0.869108	-1.89
2	10	0.421298	(0.361902, 0.432984)	0.475328	12.82
3	10	0.571870	(0.502697, 0.570099)	0.593330	3.75
4	10	0.571870	(0.542950, 0.607079)	0.592867	3.67
5	10	0.807232	(0.777385, 0.817837)	0.806672	0.07

Table 3. 100x Probability of Queueing Data for Different Voice and Data Loads, Voice Activity Factors, and Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$

		$\beta/(\alpha+\beta) = 0.4$			$\beta/(\alpha+\beta) = 0.8$			$\beta/(\alpha+\beta) = 1.0$		
Voice	Data	$\bar{Q}$			$\bar{Q}$			$\bar{Q}$		
Load $\bar{\rho}^v$	Load $\bar{\rho}^d$	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
0.1	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	2.5	0.561	0.563	0.36	0.729	0.730	0.14	0.827	0.827	0.00
0.1	4.0	6.319	6.321	0.03	7.313	7.315	0.03	7.852	7.852	0.00
1.0	0.5	0.016	0.023	43.75	0.719	0.743	3.34	2.202	2.250	2.18
1.0	2.5	3.658	3.710	1.42	14.022	14.077	0.39	22.102	22.102	0.00
1.0	4.0	18.994	19.096	0.53	41.084	41.051	-0.08	52.352	52.223	-0.25
2.0	0.5	0.169	0.184	8.88	5.934	5.875	-0.99	17.536	17.368	-0.25
2.0	2.5	9.704	9.579	-1.29	41.319	40.400	-2.24	59.504	58.066	-2.42
2.0	4.0	34.945	33.929	-2.91	73.337	72.121	-1.66	84.803	83.551	-1.48
3.5	0.5	0.430	0.412	-4.18	13.750	13.344	-2.95	39.469	38.276	-3.02
3.5	2.5	15.318	14.991	-2.13	62.819	61.211	-2.56	82.570	80.611	-2.37
3.5	4.0	46.638	45.838	-1.72	87.403	72.121	-1.46	95.741	94.775	-1.01
5.5	0.5	0.625	0.597	-4.48	19.891	19.298	-2.98	55.611	54.038	-2.38
5.5	2.5	18.868	18.491	-1.99	74.061	72.677	-1.87	91.884	90.556	-1.15
5.5	4.0	53.027	52.286	-1.40	94.322	93.527	-0.84	98.617	98.172	-0.45
10.	0.5	0.865	0.804	-7.05	26.723	25.830	-3.34	70.170	70.262	-2.64
10.	2.5	22.286	21.816	-2.11	83.032	81.898	-1.36	97.437	96.739	-0.72
10.	4.0	58.569	57.815	-1.28	97.608	97.203	-0.41	99.761	99.624	-0.14

Table 4a. 100x Probability of Queueing Data, for Voice Loads $\bar{\rho}^v = \rho_p^v = 0.1$, for Different Data Loads and Voice Activity Factors, and Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$

		$\beta/(\alpha+\beta) = 0.4$			$\beta/(\alpha+\beta) = 0.8$			$\beta/(\alpha+\beta) = 1.0$		
Data	Data	Q_l			Q_l			Q_l		
Link l	Load ρ_l^d	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
2	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
3	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
4	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
5	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
6	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
7	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
8	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
9	0.5	0.00	0.00	–	0.00	0.00	–	0.00	0.00	–
1	2.5	0.514	0.515	0.19	0.616	0.617	0.16	0.673	0.673	0.00
2	2.5	0.514	0.515	0.19	0.616	0.617	0.16	0.673	0.673	0.00
3	2.5	0.605	0.606	1.56	0.822	0.822	0.00	0.946	0.946	0.00
4	2.5	0.562	0.564	0.35	0.734	0.734	0.00	0.833	0.834	0.12
5	2.5	0.562	0.564	1.31	0.734	0.734	0.00	0.833	0.834	0.12
6	2.5	0.980	0.980	0.00	0.834	0.834	0.00	0.980	0.980	0.00
7	2.5	0.562	0.564	1.31	0.734	0.734	0.00	0.833	0.834	0.12
8	2.5	0.562	0.564	1.31	0.734	0.734	0.00	0.833	0.834	0.12
9	2.5	0.562	0.564	1.31	0.734	0.734	0.00	0.833	0.834	0.12
1	4.0	6.013	6.015	0.03	6.649	6.650	0.02	6.986	6.986	0.00
2	4.0	6.013	6.015	0.03	6.649	6.650	0.02	6.986	6.986	0.00
3	4.0	6.601	6.603	0.03	7.882	7.883	0.02	8.566	8.566	0.00
4	4.0	6.324	6.326	0.03	7.334	7.336	0.03	7.885	7.885	0.00
5	4.0	6.324	6.326	0.03	7.334	7.336	0.03	7.885	7.885	0.00
6	4.0	6.624	6.625	0.015	7.971	7.972	0.02	8.705	8.706	0.00
7	4.0	6.324	6.324	0.00	7.334	7.336	0.03	7.885	7.885	0.00
8	4.0	6.324	6.326	0.03	7.334	7.336	0.03	7.885	7.885	0.00
9	4.0	6.324	6.326	0.03	7.334	7.336	0.03	7.885	7.885	0.00

Table 4b. 100x Probability of Queueing Data, for Voice Loads $\bar{\rho}^v = \rho_p^v = 1.0$, for Different Data Loads and Voice Activity Factors, and Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$

		$\beta/(\alpha+\beta) = 0.4$			$\beta/(\alpha+\beta) = 0.8$			$\beta/(\alpha+\beta) = 1.0$		
Data	Data	Q_l			Q_l			Q_l		
Link l	Load ρ_l^d	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	0.5	0.002	0.006	200	0.130	0.143	10.00	0.410	0.430	4.88
2	0.5	0.002	0.006	200	0.130	0.143	10.00	0.410	0.430	4.88
3	0.5	0.019	0.033	73.68	0.889	0.921	3.60	2.719	2.780	2.24
4	0.5	0.018	0.028	55.55	0.795	0.824	3.65	2.446	2.511	2.66
5	0.5	0.018	0.028	55.55	0.795	0.824	3.65	2.446	2.511	2.66
6	0.5	0.031	0.018	41.94	1.348	1.395	2.13	4.052	4.052	0.00
7	0.5	0.018	0.028	55.55	0.824	1.359	64.93	2.446	2.551	4.29
8	0.5	0.018	0.028	55.55	0.795	0.824	3.65	2.446	2.551	4.29
9	0.5	0.018	0.028	55.55	0.795	0.795	0.00	2.446	2.551	4.29
1	2.5	2.073	2.115	2.03	6.659	6.730	1.07	10.486	10.581	0.91
2	2.5	2.073	2.115	2.03	6.659	6.730	1.07	10.486	10.581	0.91
3	2.5	4.479	4.549	1.56	17.166	17.217	0.29	26.923	26.862	-0.22
4	2.5	3.821	3.821	0.00	15.031	15.080	0.33	26.886	23.831	-11.36
5	2.5	3.821	3.871	1.31	15.031	15.080	0.33	23.886	23.831	-0.23
6	2.5	5.188	5.260	1.39	20.560	20.641	0.39	31.856	31.859	0.00
7	2.5	3.821	3.871	1.31	15.031	15.080	0.33	23.886	23.831	-11.36
8	2.5	3.821	3.871	1.31	15.031	15.081	0.33	23.886	23.831	-11.36
9	2.5	3.821	3.871	1.31	15.031	15.080	0.33	23.886	23.831	-11.36
1	4.0	13.490	13.606	0.86	26.953	26.988	0.13	34.960	34.904	-0.16
2	4.0	13.490	13.606	0.86	26.953	26.953	0.00	34.960	34.907	-0.15
3	4.0	22.250	22.353	0.46	48.131	48.131	0.00	60.682	60.524	-0.32
4	4.0	19.525	19.619	0.48	43.041	42.917	-0.29	55.032	54.856	-0.32
5	4.0	19.525	19.619	0.48	43.041	42.917	-0.29	55.032	54.856	-0.32
6	4.0	24.089	24.207	0.49	52.516	52.556	0.07	65.402	65.393	-0.01
7	4.0	19.525	19.619	0.48	43.041	42.917	-0.29	55.032	54.856	-0.32
8	4.0	19.525	19.619	0.48	43.041	42.917	-0.29	55.032	54.856	-0.32
9	4.0	19.525	19.619	0.48	43.041	42.917	-0.29	55.032	54.856	-0.32

Table 4c. 100x Probability of Queueing Data, for Voice Loads $\bar{\rho}^v = \rho^v_p = 2.0$, for Different Data Loads and Voice Activity Factors, and Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$

		$\beta/(\alpha+\beta)=0.4$			$\beta/(\alpha+\beta)=0.8$			$\beta/(\alpha+\beta)=1.0$		
Data	Data	Q_l			Q_l			Q_l		
Link l	Load ρ^d_l	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	0.5	0.193	0.095	-50.78	1.688	1.727	2.31	5.085	5.253	3.30
2	0.5	0.191	0.095	-50.28	1.688	1.727	2.31	5.085	5.253	3.30
3	0.5	0.437	0.354	-18.99	7.510	7.392	-2.41	21.926	21.562	-1.66
4	0.5	0.360	0.279	-22.50	6.418	6.333	-1.32	19.231	18.958	-1.42
5	0.5	0.360	0.279	-22.50	6.418	6.333	-1.32	19.231	18.958	-1.42
6	0.5	0.560	0.495	-11.61	10.430	10.368	-0.59	29.570	29.461	-0.37
7	0.5	0.360	0.279	-22.50	6.418	6.333	-1.32	19.231	18.958	-1.42
8	0.5	0.360	0.279	-22.50	6.418	6.331	-1.35	19.231	18.952	-1.45
9	0.5	0.360	0.279	-22.50	6.418	6.333	-1.32	19.231	18.956	-1.43
1	2.5	6.950	6.836	-1.64	22.198	21.868	-1.49	34.654	33.990	-1.92
2	2.5	6.950	6.836	-1.64	22.198	21.868	-1.49	34.654	33.990	-1.92
3	2.5	15.750	14.820	-5.84	48.730	48.559	-2.35	69.777	68.087	-2.92
4	2.5	12.493	12.140	-2.82	44.225	42.972	-2.83	63.989	62.043	-3.04
5	2.5	12.493	12.140	-2.82	44.225	42.972	-2.83	63.989	62.043	-3.04
6	2.5	17.381	17.331	-0.29	56.619	56.462	-0.28	76.507	76.330	-0.23
7	2.5	12.493	12.140	-2.83	44.225	42.970	-2.92	63.989	62.041	-3.04
8	2.5	12.493	12.135	-2.87	44.225	42.965	-2.85	63.989	62.034	-3.04
9	2.5	12.493	12.140	-2.82	44.225	42.970	-2.92	63.989	62.041	-3.04
1	4.0	28.411	27.943	-1.65	53.795	52.836	-1.78	67.209	65.870	-1.98
2	4.0	28.411	27.943	-1.65	53.795	52.836	-1.78	67.209	65.879	-1.98
3	4.0	47.734	46.904	-1.74	82.588	81.412	-1.42	92.433	91.445	-1.07
4	4.0	41.161	40.244	-2.23	76.716	75.168	-2.02	88.363	86.855	-1.71
5	4.0	41.161	40.244	-2.23	76.716	75.168	-2.02	88.363	86.855	-1.71
6	4.0	51.141	51.031	-0.22	86.280	86.172	-0.13	94.567	94.487	-0.08
7	4.0	41.161	40.243	-2.23	76.716	75.166	-2.02	88.363	86.854	-1.71
8	4.0	41.161	40.234	-2.25	76.716	75.162	-2.02	88.363	86.850	-1.71
9	4.0	41.161	40.243	-2.23	76.716	75.166	-2.02	88.363	86.854	-1.71

Table 5. Probability of Voice Blocking for Different Traffic Loads, Node Transceiver Vector $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$, Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$, and No Admission Control

Offered Voice Load $\bar{\rho}^v$	$\bar{B}$ Exact Value	Knapsack Approximation	Error %
0.1	0.000292	0.000327	11.98
0.5	0.050059	0.058689	17.24
1.0	0.195044	0.215835	10.66
2.0	0.417121	0.442862	6.17
3.0	0.541520	0.569988	5.62
4.0	0.619669	0.649299	4.78
5.0	0.673842	0.703402	4.39
6.0	0.713922	0.742704	4.03
7.0	0.744934	0.772556	3.71
8.0	0.769724	0.796051	3.42
10.0	0.807006	0.830684	2.93
15.0	0.862375	0.880563	2.11

Table 6. 100x Probability of Queueing Data for Various Voice and Data Loads, Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$ and Node Transceiver Vector $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$

Offered Voice Load $\bar{\rho}^v$	Offered Data Load $\bar{\rho}^d$	$\bar{Q}$ Exact Value	Knapsack Approximation	Percent Error (%)
0.1	0.2	0.033	0.034	3.03
0.1	1.0	2.860	2.860	0.00
0.1	2.0	20.796	19.687	-5.33
1.0	0.2	1.636	1.585	-3.12
1.0	1.0	14.590	14.202	-2.66
1.0	2.0	45.702	44.874	-1.81
5.0	0.2	4.762	4.505	-5.40
5.0	1.0	27.737	26.754	-3.54
5.0	2.0	64.540	63.299	-1.92
10.0	0.2	5.773	5.411	-6.27
10.0	1.0	31.143	29.915	-3.94
10.0	2.0	68.419	67.173	-1.82

Table 7a. 100x Probability of Queueing Data at Each Link for Voice Loads $\bar{\rho}^v = \rho^v_p = 0.1$, Different Data Loads, $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ^d_l	Q_l Exact Value	Knapsack Approximation	Error %
1	0.2	0.020	0.021	5.00
2	0.2	0.020	0.021	5.00
3	0.2	0.042	0.043	2.38
4	0.2	0.033	0.034	3.03
5	0.2	0.033	0.034	3.03
6	0.2	0.046	0.047	2.17
7	0.2	0.033	0.034	3.03
8	0.2	0.033	0.034	3.03
9	0.2	0.033	0.034	3.03
1	1.0	2.527	2.528	0.04
2	1.0	2.527	2.528	0.04
3	1.0	3.154	3.155	0.03
4	1.0	2.867	2.867	0.00
5	1.0	2.867	2.867	0.00
6	1.0	3.194	3.194	0.00
7	1.0	2.867	2.867	0.00
8	1.0	2.867	2.867	0.00
9	1.0	2.867	2.867	0.00
1	2.0	18.538	18.540	0.01
2	2.0	18.538	18.540	0.01
3	2.0	20.747	20.749	0.01
4	2.0	19.703	19.705	0.01
5	2.0	19.703	19.705	0.01
6	2.0	20.825	20.827	0.05
7	2.0	19.703	19.705	0.01
8	2.0	19.703	19.705	0.01
9	2.0	19.703	19.705	0.01

Table 7b. 100x Probability of Queueing Data at Each Link for Voice Loads $\bar{\rho}^v = \rho_p^v = 1.0$, Different Data Loads, $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ_l^d	Q_l Exact Value	Knapsack Approximation	Error %
1	0.2	0.820	0.796	-2.93
2	0.2	0.820	0.796	-2.93
3	0.2	2.219	2.130	-4.01
4	0.2	1.645	1.589	-3.40
5	0.2	1.645	1.589	-3.40
6	0.2	2.639	2.599	-1.52
7	0.2	1.645	1.589	-3.40
8	0.2	1.645	1.589	-3.40
9	0.2	1.645	1.589	-3.40
1	1.0	9.928	9.676	-2.54
2	1.0	9.928	9.676	-2.54
3	1.0	18.381	17.832	-2.99
4	1.0	14.670	14.226	-3.02
5	1.0	14.670	14.226	-3.02
6	1.0	19.721	19.524	-0.99
7	1.0	14.670	14.226	-3.02
8	1.0	14.670	14.203	-3.18
9	1.0	14.670	14.226	-3.02
1	2.0	37.127	36.479	-1.75
2	2.0	37.127	36.479	-1.75
3	2.0	53.274	52.282	-1.86
4	2.0	45.880	44.920	-2.09
5	2.0	45.880	44.920	-2.09
6	2.0	54.387	54.070	-0.58
7	2.0	45.880	44.920	-2.09
8	2.0	45.880	44.879	-2.18
9	2.0	45.880	44.920	-2.09

Table 7c. 100x Probability of Queueing Data at Each Link for Voice Loads $\bar{\rho}^v = \rho_p^v = 5.0$, Different Data Loads, $\underline{T} = (4,4,4,4,4,4,4,4,4,4,4)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ_l^d	Q_l Exact Value	Knapsack Approximation	Error %
1	0.2	3.560	3.202	-10.06
2	0.2	3.560	3.202	-10.06
3	0.2	7.074	6.263	-11.46
4	0.2	4.285	4.215	-1.63
5	0.2	4.285	4.216	-1.61
6	0.2	7.235	6.909	-4.50
7	0.2	4.285	4.213	-1.68
8	0.2	4.285	4.113	-4.01
9	0.2	4.285	4.213	-1.68
1	1.0	23.141	21.667	-6.37
2	1.0	23.141	21.667	-6.37
3	1.0	37.104	34.331	-7.47
4	1.0	25.917	25.615	-1.16
5	1.0	25.917	25.617	-1.16
6	1.0	36.666	35.430	-3.37
7	1.0	25.917	25.608	-1.19
8	1.0	25.917	25.238	-2.62
9	1.0	25.917	25.609	-1.19
1	2.0	58.607	56.549	-3.51
2	2.0	58.607	56.549	-3.51
3	2.0	76.910	73.960	-3.84
4	2.0	62.263	61.840	-0.68
5	2.0	62.263	61.840	-0.68
6	2.0	75.419	73.927	-1.98
7	2.0	62.263	61.832	-0.69
8	2.0	62.263	61.360	-1.45
9	2.0	62.263	61.832	-0.69

Table 7d. 100x Probability of Queueing Data at Each Link for Voice Loads $\bar{\rho}^v = \rho_p^v = 10.0$, Different Data Loads, $\underline{T} = (4,4,4,4,4,4,4,4,4,4)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ_l^d	Q_l Exact Value	Q_l Knapsack Approximation	Q_l Error %
1	0.2	4.684	4.193	-10.48
2	0.2	4.684	4.192	-10.50
3	0.2	8.786	7.567	-13.87
4	0.2	5.023	4.958	-1.29
5	0.2	5.023	4.958	-1.29
6	0.2	8.694	8.110	-6.72
7	0.2	5.022	4.956	-1.31
8	0.2	5.022	4.812	-4.18
9	0.2	5.022	4.956	-1.31
1	1.0	27.294	25.533	-6.45
2	1.0	27.294	25.529	-6.47
3	1.0	42.279	38.508	-9.39
4	1.0	28.449	28.223	-0.79
5	1.0	28.449	28.225	-0.79
6	1.0	41.173	39.049	-5.16
7	1.0	28.449	28.218	-0.81
8	1.0	28.449	27.731	-2.52
9	1.0	28.449	28.219	-3.40
1	2.0	63.910	61.736	-3.40
2	2.0	63.910	61.732	-3.41
3	2.0	81.697	78.150	-4.34
4	2.0	65.276	65.007	-0.41
5	2.0	65.276	65.009	-0.41
6	2.0	79.873	77.490	-2.98
7	2.0	65.276	65.002	-0.42
8	2.0	65.276	65.430	0.24
9	2.0	65.276	65.002	-0.42

Table 8. Probability of Voice Blocking for Different Traffic Loads

Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$

Voice Transceiver Vector $\underline{T}^v = (5,7,7,7,4,7,4,7,7,7)$

Data Link Capacity Vector $\underline{c}^d = (1,1,1,1,1,1,1,1,1)$

Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice Path p	Offered Voice Load ρ_p^v	B_p			
		Exact Value	Monte Carlo Summation	Knapsack Approximation	Error %
1	2.5	0.634	(0.673, 0.686)	0.705	11.19
2	2.5	0.144	(0.133, 0.150)	0.147	2.08
3	2.5	0.429	(0.416, 0.433)	0.416	-3.03
4	2.5	0.429	(0.416, 0.433)	0.416	-3.03
5	2.5	0.641	(0.627, 0.642)	0.658	2.65
Average	2.5	0.465	(0.453, 0.69)	0.468	0.65

Table 9. Average Data Delay for Different Voice and Data Loads
Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$
Voice Transceiver Vector $\underline{T}^v = (5,7,7,7,4,7,4,7,7,7)$
Data Link Capacity Vector $\underline{c}^d = (1,1,1,1,1,1,1,1,1)$
Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Offered Voice Load $\bar{\rho}^v$	Offered Data Load $\bar{\rho}^d$	$\bar{W}$ Exact Value	Knapsack Approximation	Error %
2.5	0.7	0.021	0.023	9.52
2.5	0.9	0.062	0.071	14.51
2.5	0.999	5.050	5.806	14.97
10.	0.7	0.034	0.036	5.88
10.	0.9	0.108	0.111	2.78
10.	0.999	9.324	9.599	2.95

Table 10a. Average Data Delay at Each Link for Voice Loads $\bar{\rho}^v = \rho_p^v = 2.5$, Different Data Loads, $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$, $\underline{T}^v = (5,7,7,7,4,7,4,7,7,7)$, $\underline{c}^d = (1,1,1,1,1,1,1,1,1)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ_l^d	W_l Exact Value	W_l Knapsack Approximation	W_l Error %
1	0.7	0.004	0.006	50.00
2	0.7	0.004	0.006	50.00
3	0.7	0.026	0.028	7.69
4	0.7	0.023	0.026	13.04
5	0.7	0.023	0.026	13.04
6	0.7	0.039	0.039	0.00
7	0.7	0.023	0.026	13.04
8	0.7	0.023	0.026	13.04
9	0.7	0.023	0.026	13.04
1	0.9	0.011	0.017	54.55
2	0.9	0.011	0.017	54.55
3	0.9	0.077	0.086	11.69
4	0.9	0.068	0.079	16.18
5	0.9	0.068	0.079	16.18
6	0.9	0.119	0.120	0.84
7	0.9	0.068	0.079	16.18
8	0.9	0.068	0.079	16.18
9	0.9	0.068	0.079	16.18
1	0.999	0.743	1.149	54.64
2	0.999	0.743	1.149	54.64
3	0.999	6.210	7.097	14.28
4	0.999	5.516	6.512	18.06
5	0.999	5.516	6.512	18.06
6	0.999	10.175	10.297	1.20
7	0.999	5.516	6.512	18.06
8	0.999	5.516	6.511	18.04
9	0.999	5.516	6.512	18.04

Table 10b. Average Data Delay at Each Link for Voice Loads $\bar{\rho}^v = \rho_p^v = 10.0$, Different Data Loads, $\underline{T} = (8,8,8,8,8,8,8,8,8,8,8)$, $\underline{T}^v = (5,7,7,7,4,7,4,7,7,7)$, $\underline{c}^d = (1,1,1,1,1,1,1,1,1)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Data Load ρ_l^d	W_l Exact Value	Knapsack Approximation	Error %
1	0.7	0.013	0.013	0.00
2	0.7	0.013	0.013	0.00
3	0.7	0.046	0.046	0.00
4	0.7	0.035	0.037	5.71
5	0.7	0.035	0.037	5.71
6	0.7	0.063	0.063	0.00
7	0.7	0.035	0.037	5.71
8	0.7	0.035	0.037	5.71
9	0.7	0.035	0.037	5.71
1	0.9	0.039	0.039	0.00
2	0.9	0.039	0.039	0.00
3	0.9	0.143	0.142	-0.70
4	0.9	0.110	0.116	5.45
5	0.9	0.110	0.116	5.45
6	0.9	0.202	0.200	-0.99
7	0.9	0.110	0.116	5.45
8	0.9	0.110	0.115	4.45
9	0.9	0.110	0.116	5.45
1	0.999	3.032	3.029	-0.09
2	0.999	3.032	3.029	-0.09
3	0.999	12.390	12.350	-0.32
4	0.999	9.458	10.008	5.82
5	0.999	9.458	10.009	5.83
6	0.999	18.173	18.007	-0.91
7	0.999	9.458	10.006	5.79
8	0.999	9.458	9.949	5.19
9	0.999	9.458	10.006	5.79

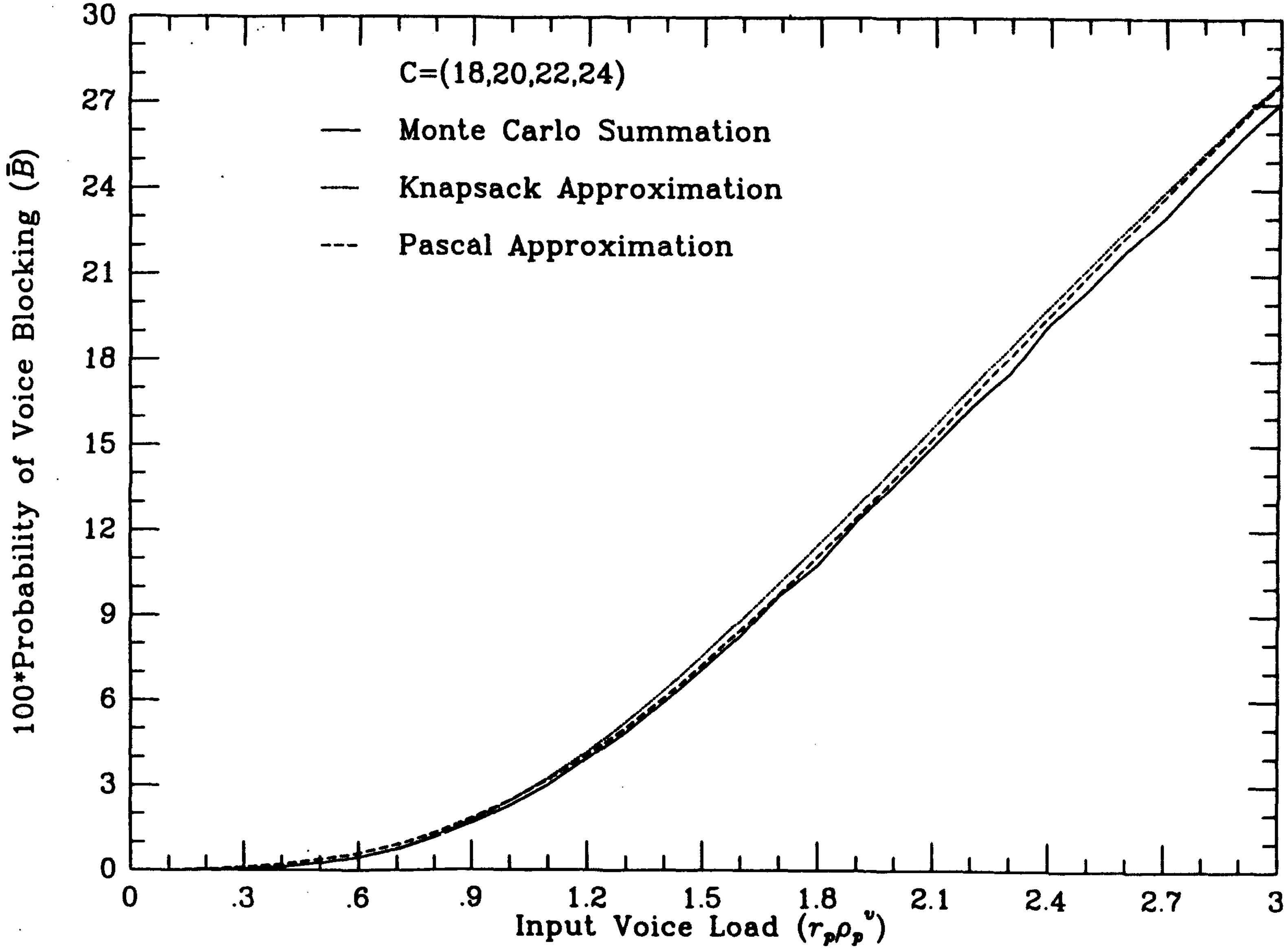

Figure 6. Approximations to the probability of voice blocking versus the offered voice load for the multi-rate network model: no admission control

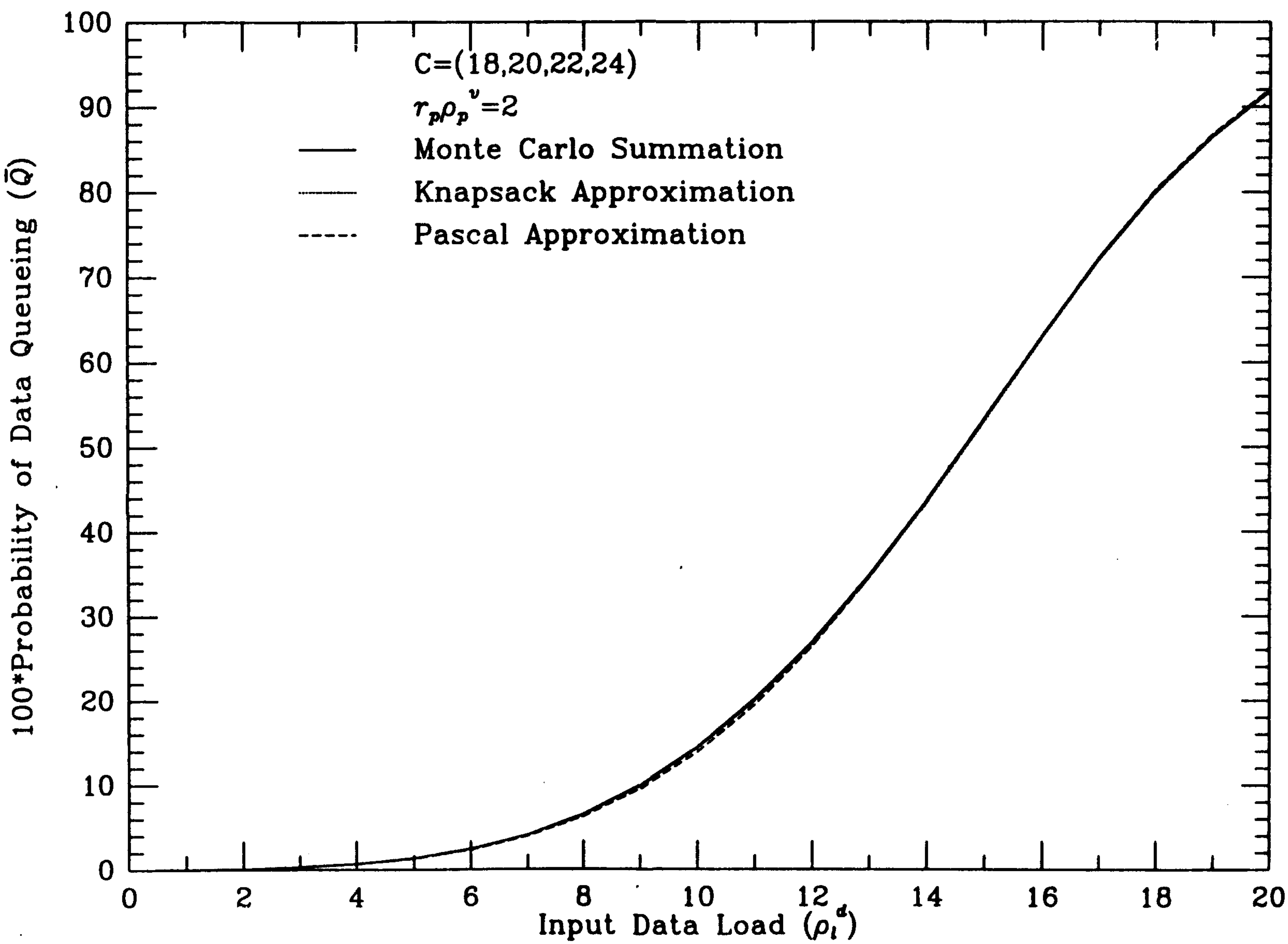

Figure 7. Approximations to the probability of data queueing versus the offered data load for the multi-rate network model: no admission control

Table 11. Percentage of Voice Calls Blocked for Different Capacity Allocations in the Multi-Rate Network of [6]

Capacity Allocation 1 $\underline{c} = (90, 100, 110, 120)$

Capacity Allocation 2 $\underline{c} = (18, 20, 22, 24)$

Capacity Allocation 3 $\underline{c} = (9, 10, 11, 12)$

Capacity Allocation 4 $\underline{c} = (5, 5, 6, 6)$

Vector of Voice Path Rates $\underline{r} = (1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 5)$

Voice Activity Factor $\beta/(\alpha + \beta) = 0.4$

Capacity Allocation	Offered Voice Load $\bar{\rho}^v$	$\bar{B}$ Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	8.0	(0.052, 0.058)	0.061	0.072
1	10.0	(0.669, 0.732)	0.741	0.756
1	15.0	(9.947, 10.275)	10.359	10.487
2	0.3	(0.042, 0.042)	0.042	0.109
2	0.7	(0.707, 0.785)	0.779	0.921
2	1.7	(9.466, 9.686)	10.137	9.758
3	0.004	(0.049, 0.070)	0.061	0.038
3	0.04	(0.630, 0.696)	0.681	0.437
3	0.4	(9.884, 10.096)	10.613	8.857
4	0.0002	(0.046, 0.046)	0.048	0.045
4	0.003	(0.629, 0.679)	0.714	0.677
4	0.05	(9.626, 9.864)	10.584	9.999

Table 12a. Percentage of Voice Calls Blocked at Each Path for Average Voice Loads $\bar{\rho}^v$ = 8.0, 10.0, and 15.0 Capacity Allocation 1, $\underline{c}$ = $(90, 100, 110, 120)$, $\underline{r} = (1,1,1,1,1,1,5,5,5,5,5,5)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice Path p	Offered Voice Load ρ_p^v	B_p Monte Carlo Summation	B_p Knapsack Approximation	B_p Pascal Approximation
1	8.0	(0.008, 0.030)	0.028	0.034
2	8.0	(0.002, 0.034)	0.025	0.030
3	8.0	(0.002, 0.034)	0.025	0.029
4	8.0	(0.001, 0.001)	0.003	0.005
5	8.0	(0.001, 0.001)	0.003	0.004
6	8.0	(0.000, 0.004)	0.000	0.001
7	1.6	(0.194, 0.207)	0.213	0.247
8	1.6	(0.144, 0.217)	0.191	0.218
9	1.6	(0.157, 0.199)	0.189	0.214
10	1.6	(0.000, 0.048)	0.028	0.038
11	1.6	(0.000, 0.052)	0.025	0.034
12	1.6	(0.000, 0.022)	0.003	0.005
1	10.0	(0.34, 0.35)	0.36	0.36
2	10.0	(0.29, 0.30)	0.30	0.31
3	10.0	(0.28, 0.29)	0.29	0.30
4	10.0	(0.07, 0.07)	0.07	0.08
5	10.0	(0.06, 0.06)	0.06	0.07
6	10.0	(0.01, 0.01)	0.01	0.01
7	2.0	(2.29, 2.30)	2.35	2.38
8	2.0	(1.96, 1.98)	1.99	2.01
9	2.0	(1.90, 1.92)	1.92	1.93
10	2.0	(0.49, 0.50)	0.53	0.57
11	2.0	(0.43, 0.44)	0.46	0.49
12	1.6	(0.08, 0.08)	0.09	0.11

Table 12a (cont'd)

Voice Path p	Offered Voice Load ρ_p^v	B_p Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	15.0	(5.70, 5.70)	5.70	5.70
2	15.0	(4.60, 4.60)	4.60	4.60
3	15.0	(4.10, 4.20)	4.20	4.20
4	15.0	(2.40, 2.40)	2.50	2.50
5	15.0	(1.90, 1.90)	2.00	2.00
6	15.0	(0.80, 0.80)	0.90	0.90
7	3.0	(28.00, 28.10)	28.60	28.50
8	3.0	(23.30, 23.40)	23.80	23.70
9	3.0	(21.40, 21.40)	21.60	21.60
10	3.0	(13.20, 13.20)	13.80	13.70
11	3.0	(10.90, 11.00)	11.40	11.30
12	3.0	(4.80, 4.80)	5.40	5.40

Table 12b. Percentage of Voice Calls Blocked at Each Path for Average Voice Loads $\bar{\rho}^v$ = 0.3, 0.7, and 1.7, Capacity Allocation 2, $\underline{c} = (18, 20, 22, 24)$, $\underline{r} = (1,1,1,1,1,1,5,5,5,5,5,5)$ and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice Path p	Offered Voice Load ρ_p^v	B_p Monte Carlo Summation	B_p Knapsack Approximation	B_p Pascal Approximation
1	0.3	(0.003, 0.003)	0.006	0.025
2	0.3	(0.003, 0.003)	0.005	0.021
3	0.3	(0.003, 0.003)	0.004	0.019
4	0.3	(0.000, 0.000)	0.002	0.011
5	0.3	(0.000, 0.000)	0.002	0.009
6	0.3	(0.000, 0.000)	0.001	0.005
7	0.06	(0.134, 0.176)	0.148	0.339
8	0.06	(0.090, 0.130)	0.107	0.281
9	0.06	(0.078, 0.134)	0.102	0.256
10	0.06	(0.034, 0.086)	0.059	0.149
11	0.06	(0.056, 0.056)	0.054	0.123
12	0.06	(0.000, 0.023)	0.012	0.056
1	0.7	(0.166, 0.227)	0.202	0.243
2	0.7	(0.148, 0.209)	0.184	0.202
3	0.7	(0.143, 0.186)	0.167	0.182
4	0.7	(0.033, 0.085)	0.069	0.119
5	0.7	(0.030, 0.058)	0.052	0.099
6	0.7	(0.000, 0.059)	0.034	0.058
7	0.14	(2.214, 2.402)	2.410	2.708
8	0.14	(1.791, 1.966)	1.946	2.258
9	0.14	(1.616, 1.773)	1.737	2.033
10	0.14	(1.022, 1.149)	1.147	1.352
11	0.14	(0.836, 0.958)	0.937	1.125
12	0.14	(0.375, 0.464)	0.465	0.668

Table 12b (cont'd)

Voice Path p	Offered Voice Load ρ_p^v	B_p		
		Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	1.7	(3.135, 3.368)	3.422	3.284
2	1.7	(2.574, 2.778)	2.836	2.780
3	1.7	(2.255, 2.459)	2.526	2.480
4	1.7	(1.826, 2.002)	2.012	1.935
5	1.7	(1.499, 1.665)	1.700	1.633
6	1.7	(0.916, 1.046)	1.103	1.120
7	0.34	(24.198, 24.763)	25.706	24.626
8	0.34	(21.002, 21.547)	22.265	21.311
9	0.34	(19.007, 19.601)	20.065	19.215
10	0.34	(14.912, 15.391)	16.246	15.670
11	0.34	(12.699, 13.148)	13.875	13.423
12	0.34	(8.795, 9.179)	9.885	9.616

Table 13. 100x Probability of Queueing Data for Different Capacity Allocations in the Multi-Rate Network of [6]

Capacity Allocation 1 $\underline{c} = (90, 100, 110, 120)$

Capacity Allocation 2 $\underline{c} = (18, 20, 22, 24)$

Capacity Allocation 3 $\underline{c} = (9, 10, 11, 12)$

Capacity Allocation 4 $\underline{c} = (5, 5, 6, 6)$

Vector of Voice Path Rates $\underline{r} = (1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 5)$

Voice Activity Factor $\beta/(\alpha + \beta) = 0.4$

Capacity Allocation	Offered Voice Load $\bar{\rho}^v$	Offered Data Load $\bar{\rho}^d$	$\bar{Q}$ Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	8.0	39.0	(0.072, 0.078)	0.072	0.088
1	8.0	49.0	(0.933, 0.956)	0.945	0.988
1	8.0	62.0	(9.747, 9.824)	9.776	9.719
1	10.0	35.0	(0.092, 0.099)	0.094	0.107
1	10.0	43.0	(0.958, 0.982)	0.776	0.807
1	10.0	57.0	(9.643, 9.725)	9.679	9.611
1	15.0	30.0	(0.079, 0.088)	0.083	0.081
1	15.0	38.0	(0.931, 0.966)	0.948	0.931
1	15.0	50.0	(9.947, 10.275)	10.106	10.080
2	0.3	6.0	(0.069, 0.074)	0.071	0.123
2	0.3	9.0	(0.800, 0.817)	0.806	0.806
2	0.3	14.0	(13.693, 13.763)	13.724	13.584
2	0.7	4.0	(0.076, 1.083)	0.081	0.136
2	0.7	7.5	(0.985, 1.012)	1.006	1.062
2	0.7	12.0	(10.302, 10.387)	10.353	9.818
2	1.7	2.0	(0.089, 0.098)	0.091	0.102
2	1.7	5.0	(1.084, 1.116)	1.079	1.068
2	1.7	10.0	(12.222, 12.335)	12.144	11.693

Table 13 (cont'd)

Capacity Allocation	Offered Voice Load $\bar{\rho}^v$	Offered Data Load $\bar{\rho}^d$	$\bar{Q}$		
			Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
3	0.004	2.0	(0.080, 0.084)	0.013	0.016
3	0.004	4.0	(1.676, 1.703)	0.901	0.888
3	0.004	6.5	(15.230, 15.290)	12.866	12.864
3	0.04	1.8	(0.091, 0.095)	0.051	0.073
3	0.04	3.6	(1.326, 1.353)	0.822	0.737
3	0.04	6.0	(11.691, 11.759)	9.621	9.536
3	0.4	0.3	(0.059, 0.067)	0.070	0.119
3	0.4	2.0	(1.071, 1.099)	1.053	1.007
3	0.4	5.0	(11.887, 11.996)	10.929	10.409
4	0.0002	0.8	(0.083, 0.086)	0.084	0.082
4	0.0002	1.4	(0.889, 0.893)	0.891	0.889
4	0.0002	2.8	(12.345, 12.348)	12.347	12.348
4	0.003	0.7	(0.104, 0.116)	0.106	0.068
4	0.003	1.4	(0.964, 0.978)	0.966	0.940
4	0.003	2.8	(12.460, 12.473)	12.463	12.475
4	0.05	0.01	(0.495, 0.525)	0.539	0.121
4	0.05	1.0	(1.248, 1.298)	1.343	0.795
4	0.05	2.6	(11.255, 11.304)	11.348	11.471

Table 14. 100x Probability of Queueing Data at Each Link for Different Voice and Data Loads, Capacity Allocation 2, $\underline{c}$ = (18, 20, 22, 24), $\underline{r}$ = (1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 5), and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Offered Voice Load $\bar{\rho}^v$	Offered Data Load ρ_l^d	Q_l Monte Carlo Summation	Q_l Knapsack Approximation	Q_l Pascal Approximation
1	0.3	6.0	(0.208, 0.226)	0.214	0.306
2	0.3	6.0	(0.049, 0.056)	0.054	0.119
3	0.3	6.0	(0.012, 0.017)	0.014	0.047
4	0.3	6.0	(0.003, 0.004)	0.003	0.019
1	0.3	9.0	(2.197, 2.247)	2.214	2.223
2	0.3	9.0	(0.719, 0.749)	0.736	0.805
3	0.3	9.0	(0.208, 0.221)	0.215	0.298
4	0.3	9.0	(0.058, 0.065)	0.060	0.114
1	0.3	14.0	(30.647, 30.793)	30.682	31.033
2	0.3	14.0	(15.219, 15.350)	15.284	14.515
3	0.3	14.0	(6.345, 6.419)	6.403	6.261
4	0.3	14.0	(2.503, 2.549)	2.527	2.528
1	0.7	4.0	(0.206, 0.226)	0.218	0.321
2	0.7	4.0	(0.068, 0.082)	0.075	0.138
3	0.7	4.0	(0.018, 0.024)	0.023	0.060
4	0.7	4.0	(0.004, 0.006)	0.007	0.026
1	0.7	7.5	(2.540, 2.616)	2.583	2.513
2	0.7	7.5	(0.957, 1.001)	0.981	1.075
3	0.7	7.5	(0.317, 0.340)	0.340	0.461
4	0.7	7.5	(0.106, 0.119)	0.119	0.199
1	0.7	12.0	(23.175, 23.380)	23.283	21.744
2	0.7	12.0	(10.997, 11.132)	11.039	10.540
3	0.7	12.0	(4.894, 4.985)	4.976	4.840
4	0.7	12.0	(2.069, 2.126)	2.113	2.147

Table 14 (cont'd)

Data Link l	Offered Voice Load $\bar{\rho}^v$	Offered Data Load ρ_l^d	Q_l		
			Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	1.7	2.0	(0.212, 0.238)	0.218	0.229
2	1.7	2.0	(0.090, 0.107)	0.098	0.107
3	1.7	2.0	(0.031, 0.040)	0.036	0.049
4	1.7	2.0	(0.011, 0.018)	0.014	0.023
1	1.7	5.0	(2.541, 2.632)	2.521	2.351
2	1.7	5.0	(1.088, 0.107)	1.104	1.130
3	1.7	5.0	(0.481, 0.520)	0.488	0.537
4	1.7	5.0	(0.184, 0.207)	0.202	0.254
1	1.7	10.0	(24.714, 24.963)	24.514	23.500
2	1.7	10.0	(13.433, 13.621)	13.383	12.965
3	1.7	10.0	(7.186, 7.331)	7.169	6.830
4	1.7	10.0	(3.443, 3.539)	3.508	3.479

Table 15. Probability of Voice Blocking for Different Traffic Loads, $\underline{T}=(8,8,8,8,8,8,8,8,8,8)$ Voice Activity Factor $\beta/(\alpha+\beta)=0.4$, and Three Types of Admission Control

(i) Equal Thresholds = 6 on Individual Voice Path Traffic

Offered Voice Load $\bar{\rho}^v$	$\bar{B}$ Exact Value	Monte Carlo Summation	Knapsack Approximation	Percent Error (%)	Thresholds
2.5	0.185545	(0.179594, 0.185808)	0.201855	8.79	(6,6,6,6,6)
3.5	0.312365	(0.302761, 0.316174)	0.331030	5.98	(6,6,6,6,6)
4.5	0.408949	(0.385203, 0.417037)	0.429108	4.93	(6,6,6,6,6)
5.5	0.482665	(0.434905, 0.514719)	0.503491	4.31	(6,6,6,6,6)
6.5	0.540419	(0.505836, 0.540619)	0.561224	3.85	(6,6,6,6,6)
7.5	0.586800	(0.546001, 0.603585)	0.607042	3.45	(6,6,6,6,6)
8.5	0.624825	(0.534647, 0.646869)	0.644257	3.11	(6,6,6,6,6)
10.	0.670493	(0.616248, 0.705557)	0.688468	2.68	(6,6,6,6,6)
15.	0.766014	(0.676021, 0.778351)	0.779502	1.76	(6,6,6,6,6)

(ii) Optimal Individual Thresholds ≤ 6

Offered Voice Load $\bar{\rho}^v$	$\bar{B}$ Exact Value	Monte Carlo Summation	Knapsack Approximation	Percent Error (%)	Thresholds
2.5	0.185512	(0.183525, 0.186201)	0.202398	9.10	(5,6,6,6,6)
3.5	0.310001	(0.308286, 0.311606)	0.317236	2.33	(2,6,6,6,5)
4.5	0.400543	(0.398409, 0.402028)	0.431677	7.77	(2,6,6,6,3)
5.5	0.469876	(0.468602, 0.471287)	0.509798	8.49	(1,6,6,6,2)
6.5	0.524440	(0.523098, 0.526017)	0.562409	7.24	(1,6,6,6,2)
7.5	0.569865	(0.568632, 0.570945)	0.613359	7.63	(1,6,6,6,1)
8.5	0.607519	(0.606093, 0.608414)	0.647777	6.62	(1,6,6,6,1)
10.	0.654345	(0.653336, 0.655637)	0.689589	5.39	(1,6,6,6,1)
15.	0.755467	(0.754448, 0.756588)	0.778174	3.00	(1,6,6,6,1)

Table 15 (cont'd)

(iii) Optimal Full Admission Controls for $x_p \leq 6$

Offered Voice Load $\bar{\rho}^v$	$\bar{B}$ Exact Value	Monte Carlo Summation	Knapsack Approximation	Percent Error (%)	Thresholds
2.5	0.185511	(0.183525, 0.186201)	0.220061	18.62	(5,6,6,6,6,8,8,7,8,8)
3.5	0.309905	(0.308566, 0.312519)	0.355329	14.66	(3,6,6,6,5,8,8,5,8,8)
4.5	0.399708	(0.399676, 0.403976)	0.451758	13.02	(2,6,6,6,4,8,8,4,8,8)
5.5	0.468216	(0.469929, 0.474508)	0.524826	12.09	(2,6,6,6,3,8,8,3,8,8)
6.5	0.522607	(0.525087, 0.529143)	0.584340	11.81	(2,6,6,6,2,8,8,2,8,8)
7.5	0.567119	(0.571885, 0.576496)	0.625008	10.21	(2,6,6,6,2,8,8,2,8,8)
8.5	0.605317	(0.610714, 0.615877)	0.658781	8.83	(2,6,6,6,2,8,8,2,8,8)
10.	0.652700	(0.658680, 0.664670)	0.699545	7.18	(2,6,6,6,2,8,8,2,8,8)
15.	0.754687	(0.757665, 0.766253)	0.785185	4.04	(2,6,6,6,2,8,8,2,8,8)

Table 16. Comparison of Exact Value and Knapsack Approximation of the Probability of Voice Blocking for Different Threshold Admission Controls, Voice Loads $\bar{\rho}^v = \rho_p^v = 2.5$, $\beta/(\alpha+\beta) = 0.4$, and $\underline{T}=(8,8,8,8,8,8,8,8,8,8)$

Voice Path p	Thresholds=(6,6,6,6,6)		Thresholds=(6,6,6,6,5)		Thresholds=(6,6,6,6,4)		Thresholds=(6,6,6,6,3)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.289575	0.316342	0.288406	0.313684	0.282039	0.306347	0.263071	0.289559
2	0.047056	0.064568	0.047098	0.064794	0.047404	0.065425	0.048638	0.066899
3	0.160361	0.168923	0.159399	0.167262	0.154592	0.162693	0.141113	0.152319
4	0.160361	0.168906	0.159399	0.167265	0.154592	0.162744	0.141113	0.152452
5	0.270387	0.290538	0.274139	0.300755	0.295341	0.329219	0.363408	0.395894
Mean	0.185548	0.201855	0.185688	0.202752	0.186794	0.205286	0.191469	0.211425

Voice Path p	Thresholds=(6,6,6,6,2)		Thresholds=(6,6,6,6,1)		Thresholds=(5,6,6,6,6)		Thresholds=(5,6,6,6,5)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.228059	0.258735	0.182889	0.214820	0.291666	0.324804	0.290500	0.322249
2	0.051652	0.069716	0.056587	0.074001	0.046530	0.063860	0.046571	0.064073
3	0.117384	0.133544	0.088578	0.107302	0.159831	0.167579	0.158866	0.165881
4	0.117384	0.133729	0.088578	0.107406	0.159831	0.167581	0.158866	0.165905
5	0.505113	0.525033	0.722943	0.729526	0.269719	0.288164	0.273474	0.298491
Mean	0.203918	0.224151	0.227915	0.246611	0.185515	0.202398	0.185655	0.203320

Voice Path p	Thresholds=(5,6,6,6,4)		Thresholds=(5,6,6,6,3)		Thresholds=(5,6,6,6,2)		Thresholds=(5,6,6,6,1)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.284161	0.315225	0.265329	0.299243	0.230826	0.270156	0.187319	0.229298
2	0.046869	0.064666	0.048063	0.066049	0.050938	0.068647	0.055425	0.072548
3	0.154044	0.161223	0.140500	0.150680	0.116583	0.131662	0.087227	0.105221
4	0.154044	0.161299	0.140506	0.140841	0.116583	0.131875	0.087227	0.103339
5	0.294691	0.327185	0.362802	0.394364	0.304595	0.523975	0.722627	0.729078
Mean	0.186762	0.205920	0.191440	0.210235	0.163905	0.225263	0.227965	0.247897

Table 17. Comparison of Exact Value and Knapsack Approximation of the Probability of Voice Blocking for Different Full Admission Controls, Voice Loads $\bar{\rho}^v = \rho_p^v = 2.5$, $\underline{T}=(8,8,8,8,8,8,8,8,8,8)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice Path p	Th=(6,6,6,6,6,8,8,8,8,8)		Th=(6,6,6,6,6,8,8,7,8,8)		Th=(6,6,6,6,6,8,8,6,8,8)		Th=(6,6,6,6,6,8,8,5,8,8)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.289575	0.331588	0.290114	0.341043	0.299332	0.358541	0.338108	0.394865
2	0.047056	0.063302	0.046979	0.062597	0.045832	0.061112	0.041908	0.058403
3	0.160361	0.192089	0.159809	0.188046	0.151728	0.179596	0.124889	0.164308
4	0.160361	0.192232	0.159808	0.188228	0.151728	0.179853	0.124889	0.164312
5	0.270387	0.309768	0.271007	0.319049	0.281622	0.338953	0.325117	0.377960
Mean	0.185548	0.217796	0.185543	0.219793	0.186048	0.223611	0.190982	0.231969

Voice Path p	Th=(6,6,6,6,5,8,8,8,8,8)		Th=(6,6,6,6,5,8,8,7,8,8)		Th=(6,6,6,6,5,8,8,6,8,8)		Th=(6,6,6,6,5,8,8,5,8,8)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.288406	0.329649	0.288937	0.338184	0.298430	0.356612	0.338108	0.393149
2	0.047098	0.063465	0.047019	0.062760	0.045855	0.061269	0.041908	0.058581
3	0.159399	0.190586	0.158921	0.186694	0.151316	0.178507	0.124889	0.163610
4	0.159399	0.190735	0.158921	0.186875	0.151316	0.178752	0.124889	0.163605
5	0.274139	0.318288	0.274612	0.327005	0.283788	0.345839	0.325117	0.383145
Mean	0.185688	0.218545	0.185682	0.220304	0.186141	0.224196	0.190982	0.232418

Voice Path p	Th=(6,6,6,6,4,8,8,8,8,8)		Th=(6,6,6,6,4,8,8,7,8,8)		Th=(6,6,6,6,4,8,8,6,8,8)		Th=(6,6,6,6,4,8,8,5,8,8)	
	B_p		B_p		B_p		B_p	
	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.	Exact Value	Knaps. Approx.
1	0.282039	0.324108	0.282489	0.332499	0.291800	0.350841	0.333782	0.387673
2	0.047404	0.063930	0.047328	0.063232	0.046101	0.061737	0.041997	0.058941
3	0.154592	0.186303	0.154254	0.182783	0.148010	0.175258	0.123861	0.161392
4	0.154592	0.186474	0.154254	0.182972	0.148010	0.175487	0.123861	0.161365
5	0.295341	0.342958	0.295602	0.350382	0.301656	0.366705	0.333778	0.399807
Mean	0.186794	0.220755	0.186785	0.222374	0.191456	0.226006	0.187115	0.233836

Table 18. 100x Probability of Queueing Data for Different Voice Loads and Thresholds, Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$, Data Loads $\bar{\rho}^d = \rho_l^d = 2.5$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Offered Voice Load $\bar{\rho}^v$	Path Thresholds	$\bar{Q}$ Exact Value	Knapsack Approximation	Percent Error (%)
2.5	(5,6,6,6,6)	12.073	11.611	-3.83
3.5	(5,6,6,6,5)	14.054	13.358	-4.95
4.5	(2,6,6,6,3)	16.012	14.794	-7.61
5.5	(1,6,6,6,2)	15.284	13.834	-9.49
6.5	(1,6,6,6,2)	16.778	15.104	-9.98
7.5	(1,6,6,6,1)	16.839	14.511	-13.82
8.5	(1,6,6,6,1)	17.897	15.357	-14.19
10.	(1,6,6,6,1)	19.110	16.345	-14.47
15.	(1,6,6,6,1)	21.397	18.474	-13.66

Table 19. 100x Probability of Queueing Data at Each Link for Data Loads $\bar{\rho}^d = \rho_l^d = 2.5$, Different Voice Loads and Control Thresholds, $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$, and Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Data Link l	Voice Load $\bar{\rho}^v = 2.5$ Thresholds=(5,6,6,6,6)			Voice Load $\bar{\rho}^v = 3.5$ Thresholds=(5,6,6,6,5)			Voice Load $\bar{\rho}^v = 4.5$ Thresholds=(2,6,6,6,3)		
	Q_l			Q_l			Q_l		
	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	6.864	6.633	-3.37	7.814	7.443	-4.75	10.232	9.325	-8.86
2	6.864	6.633	-3.37	7.814	7.444	-4.74	10.232	9.325	-8.86
3	15.103	14.573	-3.51	17.818	16.854	-5.41	20.562	18.905	-8.06
4	12.490	11.998	-3.94	14.491	13.727	-5.27	16.014	14.811	-7.51
5	12.490	11.998	-3.94	14.491	13.726	-5.28	16.014	14.810	-7.52
6	17.376	17.123	-1.46	20.586	19.864	-3.51	23.009	21.579	-6.21
7	12.490	11.998	-3.94	14.491	13.730	-5.25	16.015	14.816	-7.49
8	12.490	11.994	-3.97	14.491	13.707	-5.41	16.015	14.760	-7.84
9	12.490	11.998	-3.94	14.491	13.730	-5.25	16.015	14.816	-7.49

Data Link l	Voice Load $\bar{\rho}^v = 5.5$ Thresholds=(1,6,6,6,2)			Voice Load $\bar{\rho}^v = 6.5$ Thresholds=(1,6,6,6,2)			Voice Load $\bar{\rho}^v = 7.5$ Thresholds=(1,6,6,6,1)		
	Q_l			Q_l			Q_l		
	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	9.280	9.127	-1.65	10.390	10.217	-1.66	11.626	11.214	-3.54
2	9.280	9.127	-1.65	10.390	10.217	-1.66	11.626	11.214	-3.54
3	19.603	17.888	-8.75	21.527	19.503	-9.40	21.915	19.077	-12.95
4	15.302	13.642	-10.85	16.753	14.843	-11.40	16.329	13.783	-15.59
5	15.302	13.642	-10.85	16.753	14.843	-11.40	16.329	13.783	-15.59
6	22.885	20.317	-11.22	24.930	21.992	-11.78	24.733	20.581	-16.79
7	15.301	13.646	-10.82	16.753	14.847	-11.38	16.330	13.786	-15.58
8	15.301	13.472	-11.95	16.753	14.623	-12.71	16.330	13.371	-18.12
9	15.301	13.646	-10.82	16.753	14.847	-11.38	16.330	13.786	-15.58

Table 19 (cont'd)

Data Link l	Voice Load $\bar{\rho}^v = 8.5$ Thresholds=(1,6,6,6,1)			Voice Load $\bar{\rho}^v = 10.0$ Thresholds=(1,6,6,6,1)			Voice Load $\bar{\rho}^v = 15.0$ Thresholds=(1,6,6,6,1)		
	Q_l			Q_l			Q_l		
	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %	Exact Value	Knapsack Approx.	Error %
1	12.389	11.962	-3.45	13.268	12.869	-3.00	14.932	14.793	-0.93
2	12.389	11.962	-3.45	13.268	12.870	-3.00	14.932	14.793	-0.93
3	23.239	20.114	-13.45	24.747	21.356	-13.70	27.569	23.974	-13.04
4	17.373	14.570	-16.13	18.572	15.520	-16.43	20.842	17.534	-15.87
5	17.373	14.570	-16.13	18.572	15.520	-16.43	20.842	17.535	-15.87
6	26.189	21.654	-17.32	27.844	22.936	-17.63	30.931	25.648	-17.08
7	17.373	14.577	-16.09	18.572	15.527	-16.39	20.842	17.551	-15.79
8	17.373	14.101	-18.83	18.572	14.981	-19.33	20.842	16.888	-18.97
9	17.373	14.577	-16.09	18.572	15.528	-16.39	20.842	17.552	-15.78

Table 20. Comparison of Voice-Blocking Performance of Admission Control Policies Whose Thresholds are Obtained by Optimizing the Knapsack Approximation Node Transceiver Vector $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$ Voice Activity Factor $\beta/(\alpha+\beta) = 0.4$

Voice	No Control		Knapsack-Based Thresholds				Optimal Thresholds		
	$\bar{B}$		$\bar{B}$				$\bar{B}$		
Load $\bar{\rho}^v$	Exact Value	Knapsack Approx	Knapsack Approx	Path Thresh	Exact Value	Gain %	Exact Value	Path Thresh	Gain %
3.5	0.307931	0.317236	0.318865	(3,8,8,8,8)	0.305456	0.80	0.304054	(2,8,8,8,6)	0.83
4.5	0.401453	0.411132	0.407689	(1,8,8,8,8)	0.386530	3.72	0.385776	(1,8,8,8,3)	3.91
5.5	0.472130	0.483226	0.477817	(1,8,8,8,8)	0.453197	4.01	0.448496	(1,8,8,8,2)	5.01
6.5	0.527213	0.539849	0.534412	(1,8,8,8,4)	0.507246	3.79	0.498001	(1,8,8,8,1)	5.53
7.5	0.571417	0.585356	0.580005	(1,8,8,8,3)	0.550631	3.64	0.541157	(1,8,8,8,1)	5.29
8.5	0.607742	0.622658	0.617512	(1,8,8,8,2)	0.584775	3.78	0.578903	(1,8,8,8,1)	4.74
10.	0.651634	0.667461	0.662817	(1,8,8,8,2)	0.632611	2.92	0.626443	(1,8,8,8,1)	3.86
15.	0.745317	0.761318	0.758245	(1,8,8,8,1)	0.730823	1.94	0.730823	(1,8,8,8,1)	1.94

Table 21. Revenue Sensitivities for the Multi-Rate Network of [6]

Link Capacity Vector $\underline{c} = (90, 100, 110, 120)$

Vector of Voice Loads $\underline{\rho}^v = (10, 10, 10, 10, 10, 10, 2, 2, 2, 2, 2, 2)$

Vector of Data Loads $\underline{\rho}^d = (50, 50, 50, 50)$

Vector of Voice Rates $\underline{r} = (1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 5,)$

Vector of Voice Revenue Rates $\underline{\gamma}^v = (1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.0, 3.6, 4.2, 4.8, 5.4, 6.0)$

Vector of Data Revenue Rates $\underline{\gamma}^d = (1.0, 1.2, 1.4, 1.6)$

Voice Activity Factor $\beta/(\alpha + \beta) = 0.4$

Table 21a. Voice Revenue Sensitivity With Respect to Voice Loads

Voice Path p	Offered Voice Load ρ_p^v	$\partial W^v / \partial \rho_p^v$ Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	10.0	(0.88, 0.99)	0.90	0.90
2	10.0	(1.13, 1.24)	1.12	1.12
3	10.0	(1.27, 1.39)	1.33	1.33
4	10.0	(1.50, 1.61)	1.57	1.57
5	10.0	(1.61, 1.81)	1.77	1.77
6	10.0	(1.97, 2.09)	1.99	1.99
7	2.0	(2.19, 2.43)	2.35	2.37
8	2.0	(3.00, 3.24)	3.07	3.08
9	2.0	(3.54, 3.79)	3.69	3.70
10	2.0	(4.48, 4.73)	4.58	4.57
11	2.0	(5.19, 5.24)	5.21	5.21
12	2.0	(5.75, 6.02)	5.95	5.94

Table 21b. Data Revenue Sensitivity With Respect to Voice Loads

Voice Path p	Offered Voice Load ρ_p^v	$\partial W^d/\partial\rho_p^v$ Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	10.0	(-0.413765, -0.391275)	-0.395828	-0.385843
2	10.0	(-0.334608, -0.313041)	-0.321129	-0.314085
3	10.0	(-0.321502, -0.300196)	-0.307425	-0.299321
4	10.0	(-0.117725, -0.101117)	-0.106280	-0.106447
5	10.0	(-0.104447, -0.088175)	-0.092525	-0.091626
6	10.0	(-0.024743, -0.009671)	-0.017448	-0.019493
7	2.0	(-2.552438, -2.500914)	-2.501584	-2.444003
8	2.0	(-2.038939, -1.992859)	-1.997447	-1.960893
9	2.0	(-1.937221, -1.892167)	-1.892145	-1.852493
10	2.0	(-0.787522, -0.758234)	-0.767471	-0.759875
11	2.0	(-0.682234, -0.654666)	-0.659719	-0.648903
12	2.0	(-0.144140, -0.126442)	-0.138872	-0.149429

Table 21c. Data Revenue Sensitivity With Respect to Data Loads

Data Link l	Offered Data Load ρ_l^d	$\partial W^d/\partial\rho_l^d$ Monte Carlo Summation	Knapsack Approximation	Pascal Approximation
1	50.0	(0.000519, 0.012731)	0.130102	0.148864
2	50.0	(0.927529, 0.934309)	0.940640	0.940621
3	50.0	(1.356462, 1.359009)	1.356561	1.352288
4	50.0	(1.595059, 1.595804)	1.595164	1.593700

Table 22. Revenue Sensitivities for the Single-Rate Radio Network of [11]

Node Transceiver Vector $\underline{T} = (8, 8, 8, 8, 8, 8, 8, 8, 8, 8)$

Vector of Voice Revenue Rates $\underline{\gamma}^v = (1.0, 1.2, 1.4, 1.6, 1.8)$

Vector of Data Revenue Rates $\underline{\gamma}^d = (1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6)$

Voice Activity Factor $\beta/(\alpha + \beta) = 0.4$

Table 22a. Voice Revenue Sensitivity With Respect To Voice Loads

Voice Path p	Offered Voice Load ρ_p^v	$\partial W^v / \partial \rho_p^v$ Exact Value	Knapsack Approximation	Percent Error (%)
1	5.5	-0.276503	-0.221452	-19.91
2	5.5	0.628294	0.503593	-19.85
3	5.5	0.412164	0.373352	-9.42
4	5.5	0.455148	0.415531	-8.70
5	5.5	0.103284	0.114084	10.46

Table 22b. Data Revenue Sensitivity With Respect To Voice Loads

Vector of Data Loads $\underline{\rho}^d = (4.0, 4.0, 4.0, 4.0, 4.0, 4.0, 4.0, 4.0, 4.0)$

Voice Path p	Offered Voice Load ρ_p^v	$\partial W^d / \partial \rho_p^v$ Exact Value	Knapsack Approximation	Percent Error (%)
1	5.5	-0.313773	-0.435772	38.88
2	5.5	-0.301060	-0.376670	25.11
3	5.5	-0.096402	-0.277122	187.46
4	5.5	-0.327656	-0.427354	30.43
5	5.5	-0.262378	-0.382214	45.67

Vector of Data Loads $\underline{\rho}^d = (2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0)$

Voice Path p	Offered Voice Load ρ_p^v	$\partial W^d / \partial \rho_p^v$ Exact Value	Knapsack Approximation	Percent Error (%)
1	5.5	-0.068434	-0.089658	31.01
2	5.5	-0.056307	-0.067183	19.31
3	5.5	-0.029062	-0.059503	104.76
4	5.5	-0.073209	-0.088104	20.35
5	5.5	-0.061949	-0.081633	31.77

Table 22c. Data Revenue Sensitivity With Respect To Data Loads

Vector of Voice Loads $\underline{\rho}^v = (5.5, 5.5, 5.5, 5.5, 5.5)$

Data Link l	Offered Data Load ρ_l^d	$\partial W^d / \partial \rho_l^d$		
		Exact Value	Knapsack Approximation	Percent Error (%)
1	4.0	-0.204096	-0.189882	-6.96
2	4.0	-0.244915	-0.227802	-6.99
3	4.0	-0.499528	-0.496486	-0.61
4	4.0	-0.440876	-0.431286	-2.17
5	4.0	-0.495986	-0.485410	-2.13
6	4.0	-0.672845	-0.671192	-0.25
7	4.0	-0.606184	-0.593026	-2.17
8	4.0	-0.661292	-0.645434	-2.40
9	4.0	-0.716399	-0.718822	0.34
1	2.0	0.732983	0.742983	1.36
2	2.0	0.879579	0.891355	1.34
3	2.0	0.755747	0.774882	2.53
4	2.0	1.044098	1.052112	0.77
5	2.0	1.174610	1.183667	0.77
6	2.0	1.048758	1.052783	0.38
7	2.0	1.435631	1.446897	0.78
8	2.0	1.566143	1.580366	0.91
9	2.0	1.696655	2.079021	22.54

Table 23. Computational Effort Required for the Different Approximations and Network Models

Approximation Method	Radio Network $\underline{T} = (8,8,8,8,8,8,8,8,8,8)$ $\underline{r} = (1,1,1,1,1)$		Multi-Rate Network $\underline{c} = (90,100,110,120)$ $\underline{r} = (1,1,1,1,1,1,5,5,5,5,5,5)$	
	Probability of Voice Blocking	Probability of Data Queueing	Probability of Voice Blocking	Probability of Data Queueing
Exact	20 sec	20 sec	prohibitive	prohibitive
Monte Carlo	1 min	1 min	25 min	25 min
Knapsack	2 sec	2 sec	2.5 sec	2.5 sec
Pascal	2 sec	2 sec	2.5 sec	2.5 sec